

Parasitic Oscillations And Frequency Deviation In LC-Oscillators Using IC Amplifiers

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Abstract

While integrated circuit (IC) amplifiers are mostly used with RC and crystal oscillators, they can also be used as a feedback device for LC oscillators. However, two basic problems of practical LC oscillators, the development of parasitic oscillations with frequencies above the intended oscillation frequency range under certain conditions and a deviation of the actual oscillation frequency from the natural frequency $\omega_0 = 1/\sqrt{LC}$ of the LC tank often become more pronounced when using integrated circuit amplifiers instead of one single active component. This is mostly because integrated circuit amplifiers often exhibit an appreciable delay between input and output signal even at relatively low frequencies. In this paper, we shall investigate both, parasitic oscillations and frequency deviation, using a generic non-ideal LC oscillator model.

A Generic LC-Oscillator With IC Amplifier

We shall base our analysis on a generic LC oscillator using a differential voltage amplifier as shown in figure 1. The LC tank is modeled by an ideal inductor L , an ideal capacitor C and a parallel loss resistance R_P that subsumes the losses occurring in the inductor and the capacitor. The feedback device “FB” is a generic differential amplifier, delivering it’s output voltage U_f back into the LC tank via R_f .

In the ideal case, this differential amplifier exhibits neither a delay time between input and output nor is it’s amplification behavior frequency dependent. However, integrated circuit amplifiers, due to the relatively large number of internal transistor systems typically have an appreciable delay between input and output signal even at relatively low frequencies, resulting in a phase shift that can easily reach values of 360° or beyond. In this paper we shall therefore explicitly focus on the non-ideal case that involves an appreciable phase shift as well as a frequency dependent amplification behavior with a roll-off at higher frequencies.

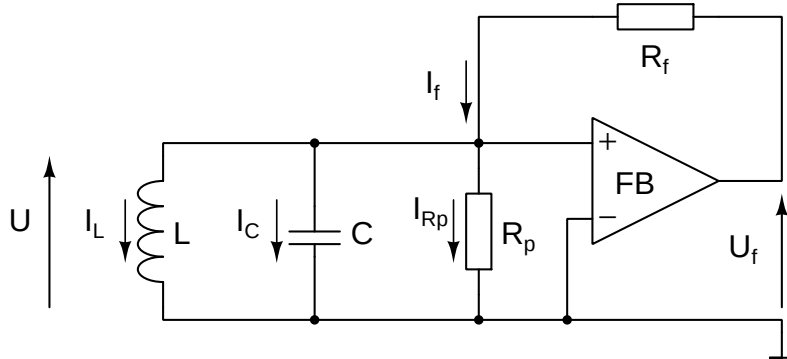


Figure 1: Generic LC oscillator

Linear Approach

A basic understanding of the behavior of the oscillator with phase shifted feedback can be obtained by going below the oscillation threshold and analyzing the response of the feedback device when driven externally by a small sinusoidal input voltage using complex phasors. We shall therefore, for now, remove the LC tank from the circuit while keeping it's parallel loss resistance R_p in place and take a look at the input current I_{IN} resulting from a driving voltage U_{IN} as depicted in figure 2.

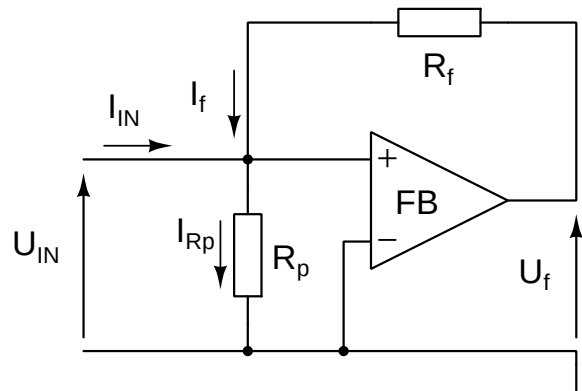


Figure 2: Feedback device in linear phasor analysis

Using Kirchhoff's circuit laws [1], we obtain

$$U_{R_f} + U_{IN} = U_f \quad \text{and} \quad I_{IN} + I_f = I_{R_p}$$

In the linear approach, the feedback voltage is related to the input voltage by $U_f = aU_{\text{IN}}$ where a is the small signal amplification factor of the feedback device. Using Ohm's law, these equations yield

$$I_{\text{IN}} = \left(\frac{1}{R_P} + \frac{1}{R_f} - \frac{a}{R_f} \right) U_{\text{IN}}$$

from which the input impedance follows to be

$$Z_{\text{IN}} = \left(\frac{1}{R_P} + \frac{1}{R_f} - \frac{a}{R_f} \right)^{-1} = R_P \parallel R_f \parallel \frac{-R_f}{a}$$

where $Z_1 \parallel Z_2$ denotes impedances connected in parallel. The fact that R_f contributes to the input impedance can easily be understood by bearing in mind that the feedback device is assumed to have zero output impedance, hence R_P and R_f are connected in parallel.

As expected, positive feedback ($a > 0$) creates a negative virtual parallel loss resistance $-R_f/a$. If this negative resistance over-compensates the losses in the tank, i.e.

$$\frac{1}{R_P} + \frac{1}{R_f} - \frac{a}{R_f} < 0$$

the circuit will start to oscillate. We can now easily introduce a phase shift φ between input and output voltage of the feedback device by using a complex amplification factor of the form $ae^{-j\varphi}$. By putting a negative sign into the exponent, we imply that the output lags behind the input and we have $\varphi > 0$ for the remainder of this paper. Replacing a with $ae^{-j\varphi}$ in the above equations immediately yields

$$Z_{\text{IN}} = \left(\frac{1}{R_P} + \frac{1}{R_f} - \frac{a \cos \varphi}{R_f} + j \frac{a \sin \varphi}{R_f} \right)^{-1} = R_P \parallel R_f \parallel \frac{-R_f}{a \cos \varphi} \parallel \frac{-jR_f}{a \sin \varphi}$$

Obviously, a phase shift φ occurring in the feedback device has two effects: First, the additional parallel resistance $-R_f/a \cos \varphi$ introduced by the feedback device now depends on the phase shift φ and can also assume positive values, i.e. the feedback device delivers negative feedback into the tank causing damping. Second, a virtual parallel reactance $-jR_f/a \sin \varphi$ is introduced into the tank causing detuning, i.e. the actual oscillation frequency deviates from the natural frequency $\omega_0 = 1/\sqrt{LC}$ of the tank. In case of $\sin \varphi > 0$ the tank will be detuned towards lower frequencies (virtual parallel capacitance) while in case of $\sin \varphi < 0$ the tank will be detuned towards higher frequencies (virtual parallel inductance).

Let us now focus on the oscillation conditions of the circuit as predicted by the linear approach. From the above equation for Z_{IN} the oscillation condition is found to be

$$\frac{1}{R_p} + \frac{1}{R_f} - \frac{a \cos \varphi}{R_f} < 0$$

In all practical feedback devices, the phase shift between input and output voltage as well as the small signal amplification factor depend on the frequency of the oscillations. Hence we are actually dealing with $\varphi(\omega)$ and $a(\omega)$. With ω increasing, $a(\omega)$ will typically decrease while $\varphi(\omega)$ increases. In fact, $\varphi(\omega)$ will go beyond 360° when the delay time between input and output exceeds one period of the input signal. A necessary condition for the circuit to oscillate is obviously $\cos \varphi(\omega) > 0$, i.e. oscillations can potentially occur at frequencies where $0^\circ < \varphi(\omega) < 90^\circ$ or $270^\circ < \varphi(\omega) < 450^\circ$ or $630^\circ < \varphi(\omega) < 810^\circ$ and so on. However, for oscillations to actually occur, $a(\omega)$ must still be high enough to fulfill the oscillation condition stated above.

The property that oscillations can occur at higher frequencies where $\varphi(\omega)$ is above 270° can e.g. be used in microwave oscillators using delay lines. However, in oscillators using an LC tank this behavior is generally undesired since it may cause oscillations near parasitic resonant frequencies of the circuit connected to the feedback device.¹ In this paper, we shall therefore regard oscillations beyond the primary oscillation range ($0^\circ < \varphi(\omega) < 90^\circ$) as undesired parasitic oscillations.

General Non-Linear Treatment

While the linear approach has certainly given us a valuable qualitative insight into the behavior of the oscillator, its quantitative applications are limited. This is simply because oscillators are non-linear by nature and a time domain analysis using differential equations is necessary. Since in the non-linear time domain approach we cannot simply put a phase shift between the input and output phasor of the feedback device as we did in the linear approach, we will need a suitable non-linear circuit model of the feedback device to continue.

Let's start by looking at the required properties of the feedback device model. First of all, the output voltage U_f of the feedback device needs to be bounded. Also, U_f should exhibit an approximately linear dependency on the input voltage for sufficiently small input voltages at low input frequencies. Therefore, a good starting point is the non-linear feedback device introduced in [2] with the current output replaced by a voltage output. What we need to do now, is to augment this feedback device model by a time delay mechanism (resulting in the desired phase shift) and low-pass properties so that the small signal amplification factor $a(\omega)$ decreases as the frequency rises. This can simply be achieved by preceding

¹When taking parasitic inductances and capacitances on the circuit board and in the components into account, the LC tank connected to the input becomes an intricate LC network with several parasitic resonant frequencies above $\omega_0 = 1/\sqrt{LC}$.

the non-linear feedback device with a cascade of n RC low-pass filters that are mutually separated by ideal² unity gain amplifiers as shown in figure 3.

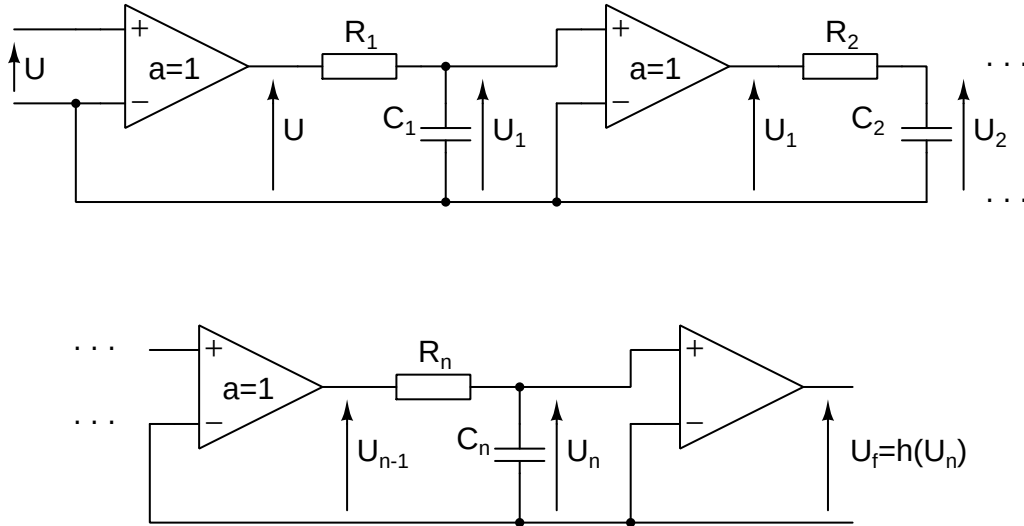


Figure 3: Non-linear feedback device with delay

As mentioned above, the last stage of this feedback device is an ideal controlled voltage source similar to the ideal controlled current source used in [2] whose output voltage U_f is given by

$$U_f = h(U_n) = a_1 \arctan(a_2 U_n)$$

and which therefore has a small signal gain of $a_0 = h'(0) = a_1 a_2$. The reader is referred to [2] for a more detailed discussion of the properties of this feedback function.

Phase Shift And Gain

Before we start deriving the differential equations governing the generic LC oscillator from figure 1 using the feedback device model from figure 3, let us do a small signal linear analysis of this feedback device model. Using phasors, it is easily shown that for a driving voltage of frequency ω , the phase shift of each

²Infinite input impedance, zero output impedance

RC low-pass is given by $\arctan(\omega\tau_k)$ where $\tau_k = R_k C_k$ is the time constant of the RC low-pass. Since the RC low-pass filters are mutually separated by unity gain amplifiers, the overall phase shift is

$$\varphi(\omega) = \sum_{k=1}^n \arctan(\omega\tau_k)$$

Also the voltage attenuation factor of each RC low-pass filter is readily obtained to be $((\omega\tau_k)^2 + 1)^{(1/2)}$ resulting in a frequency dependent gain factor of the feedback device of

$$a(\omega) = a_0 \cdot \prod_{k=1}^n \frac{1}{\sqrt{(\omega\tau_k)^2 + 1}}$$

where a_0 is the small signal gain factor of the last stage as introduced in the previous section. The overall small signal phase shift and relative gain factor $a(\omega)/a_0$ for $\tau_k = \tau = 9\text{ns}$ and $n = 20$ for a frequency range between 100kHz and 10MHz have been plotted in figure 4.

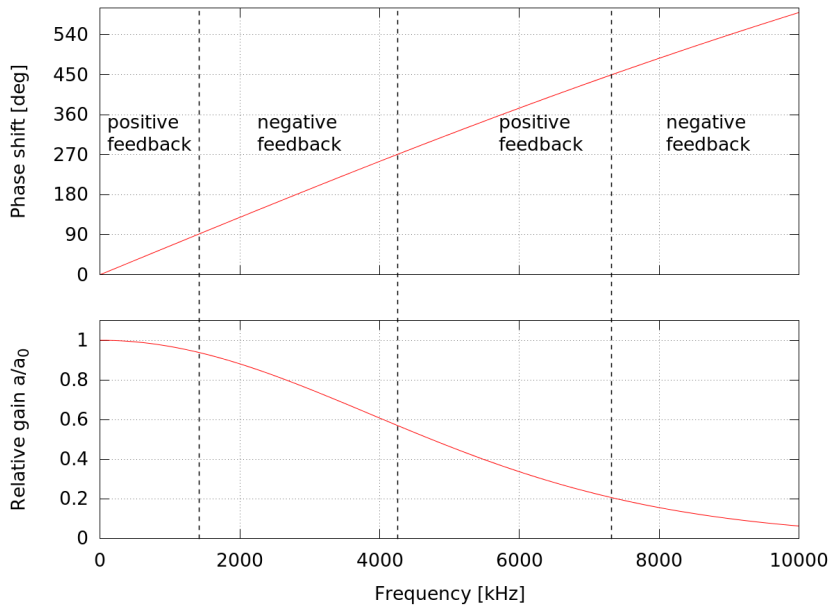


Figure 4: Phase shift and relative gain

The frequency ranges where the feedback device, according to the phase shift φ in the linear analysis, delivers positive resp. negative feedback into the LC tank have been marked accordingly in the diagram. Let us at this point recall the linear analysis of the oscillation condition: The circuit can oscillate in those frequency ranges where positive feedback is delivered into the LC tank **and** the gain $a(\omega)$ of

the feedback device is still sufficiently high to lead to a slight over-compensation of the losses in the tank. Hence, for too high a small signal, low frequency gain factor $a(\omega \rightarrow 0) = a_0$, the circuit will not only be able to oscillate in the primary (intended) frequency range from approximately 100kHz to 1200kHz, but will also be prone to parasitic oscillations in the positive feedback frequency range around 6000kHz as can be seen from figure 4

Time Domain Analysis

Let's take a closer look at the generic LC oscillator from figure 1 using the feedback device model given in figure 3. Using Kirchhoff's laws [1], we obtain

$$\begin{aligned} I_f(t) &= I_L(t) + I_C(t) + I_{R_P}(t) \\ U_f(t) &= h(U_n(t)) = U_{R_f}(t) + U_{R_P}(t) \\ U(t) &= U_L(t) = U_C(t) = U_{R_P}(t) \end{aligned}$$

as well as

$$\begin{aligned} U(t) &= U_{R_1}(t) + U_1(t) \\ U_1(t) &= U_{R_2}(t) + U_2(t) \\ &\vdots \\ U_{n-1}(t) &= U_{R_n}(t) + U_n(t) \end{aligned}$$

Using $I_{R_k}(t) = C_k \dot{U}_k(t)$ the second set of equations is readily rewritten as

$$\begin{aligned} U(t) &= U_1(t) + \tau_1 \dot{U}_1(t) \\ U_1(t) &= U_2(t) + \tau_2 \dot{U}_2(t) \\ &\vdots \\ U_{n-1}(t) &= U_n(t) + \tau_n \dot{U}_n(t) \end{aligned} \tag{1}$$

with $\tau_k = R_k C_k$. Also, differentiating the first equation in the first set with respect to time yields $\dot{I}_L(t) + \dot{I}_C(t) + \dot{I}_{R_P}(t) - \dot{I}_f(t) = 0$ which immediately leads to

$$\frac{1}{L}U(t) + C\ddot{U}(t) + \frac{1}{R_P}\dot{U}(t) - \frac{d}{dt}\left(\frac{U_{R_f}(t)}{R_f}\right) = 0$$

Using $U_{R_f}(t) = h(U_n(t)) - U_{R_P}(t) = h(U_n(t)) - U(t)$ from the first set and bearing in mind that

$$\frac{d}{dt}h(U_n(t)) = h'(U_n(t)) \cdot \dot{U}_n(t)$$

we finally arrive at

$$\frac{1}{\omega_0^2}\ddot{U}(t) + \frac{1}{\omega_0^2 C} \left(\frac{1}{R_p} + \frac{1}{R_f} \right) \dot{U}(t) + U(t) - \frac{1}{\omega_0^2 R_f C} h'(U_n(t)) \cdot U_n(t) = 0 \quad (2)$$

with $\omega_0 = 1/\sqrt{LC}$. Equation (2) along with equation set (1) completely govern the behavior of the circuit for all feedback functions $h(U_n)$ of the last stage of the feedback device model and any number n of RC low-pass filters. However, we shall continue to use $h(U_n) = a_1 \arctan(a_2 U_n)$ along with $n = 20$ as we did earlier in this paper.

Like with most non-linear differential equations we have to resort to a numerical analysis at this point. Let us start by setting the parameters that will not change during the remainder of this paper. These shall be $a_1 = 10$, thereby limiting the output voltage of the feedback device to approximately $\pm 15V$, $R_f = 100k\Omega$ and $\tau_k = \tau = 9ns$ ³ for all RC low-pass filters, equivalent to a cut-off frequency of approximately 18MHz which is well above the frequency range of 100kHz to 10MHz that we are going to look at.

An obvious issue when covering a large frequency range is the L/C ratio of the tank. In order to not end up with unreasonable L/C ratios, we shall use a constant ratio of $L/C = 630\mu H/nF$ over the entire frequency range⁴. Furthermore, the parallel loss resistance R_p is a rather non-intuitive quantity and it may be desirable to replace R_p by the Q-factor of the LC tank by virtue of [3] $R_p = Q\sqrt{L/C}$.

Parasitic Oscillations

We shall now focus on the occurrence of parasitic oscillations above the primary (intended) oscillation frequency range. The preceding linear analysis of the circuit has already given us an idea of what to expect in terms of parasitic oscillations. However any linear treatment of a highly non-linear system like an oscillator will typically fail to make decent quantitative predictions and we need to resort to numerically solving the non-linear differential equations governing the circuit in order to obtain reasonable quantitative results. For this purpose, we have used LSODE [4].

³The resulting phase shift behavior is roughly similar to the phase shift behavior of an LM311 comparator at frequencies below 5MHz.

⁴This obviously implies that $C = \frac{1}{\omega_0} \sqrt{\frac{C}{L}} \approx \frac{1}{\omega_0} \cdot 1.26 \cdot 10^{-3} \frac{F}{s}$ in equation (2)

From the linear analysis of the circuit, we know that the value of the small signal, low frequency gain factor a_0 of the feedback device is crucial for the occurrence or absence of parasitic oscillations. We shall therefore investigate the occurrence of oscillations and their amplitude for different values of a_0 as the natural frequency $\omega_0 = 1/\sqrt{LC}$ of the tank moves over a wide range from 100kHz to 10MHz⁵. This has been done in figure 5 by numerically solving the differential equations governing the circuit with parameters as specified so far in this paper and assuming a moderately high Q-factor of the LC tank of $Q = 80$.

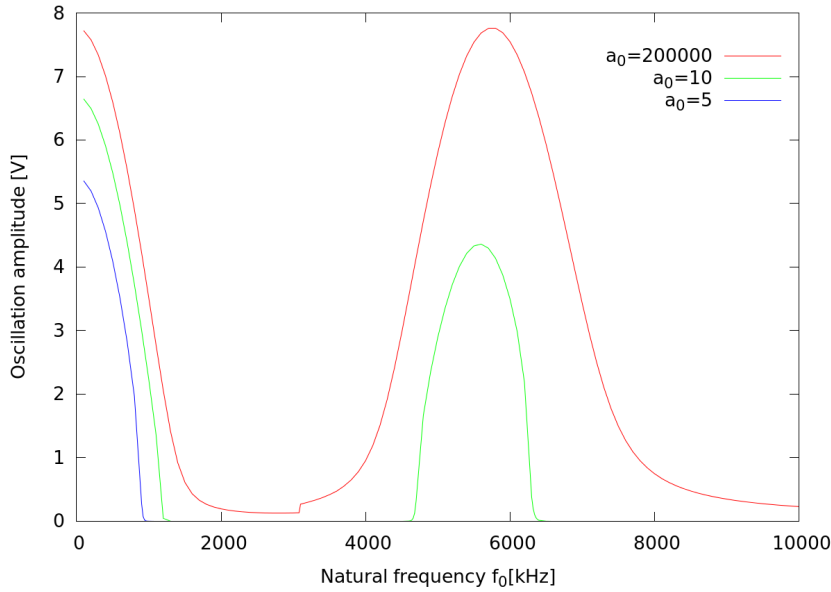


Figure 5: Occurrence of oscillations

Obviously, for a high gain of $a_0 = 200000$ ⁶ (red curve) the circuit will oscillate not only when the natural frequency of the LC tank is below approximately 1.4MHz but also when the natural frequency is in a range around 6MHz⁷. In practical oscillator circuits with the intended natural frequency of the LC tank set below 1.4MHz this may cause undesired oscillations near parasitic resonant frequencies of the circuit around 6MHz. This unfavorable behavior persists even

⁵The reader is reminded that a_0 is related to the parameters a_1 and a_2 of the feedback model by $a_0 = a_1 a_2$ and after setting a_1 can be set by choosing a_2 .

⁶This is roughly the open-loop gain of an LM311 voltage comparator.

⁷The reader may have noticed that even in the frequency range around 3MHz where the linear approach predicts no oscillations at all, the amplitude curve indicates some residual oscillations. A deeper investigation reveals non-sinusoidal oscillations with frequencies jittering between the primary and parasitic oscillation frequency range. This is clearly a non-linear phenomenon of the circuit.

down to $a_0 = 10$ (green curve) although the amplitude of the oscillations around 6MHz now becomes somewhat smaller than the amplitude of the oscillations below approximately 1100kHz. Finally, at $a_0 = 5$ (blue curve) the oscillator is left with only one oscillation frequency range ending at approximately 800kHz. This rules out parasitic oscillations above the intended frequency range and can be considered a stable oscillator design.

Frequency Deviation

We shall now turn our attention to the deviation of the oscillation frequency ω_{osc} from the natural frequency $\omega_0 = 1/\sqrt{LC}$ of the LC tank. From the linear analysis of the circuit we have learned that this is due to a virtual parallel reactance appearing across the LC tank due to feedback. Here, we are interested in the frequency deviation occurring in the primary oscillation range, where in the linear approach, $0^\circ < \varphi < 90^\circ$ and hence $\omega_{\text{osc}} < \omega_0$ due to the appearance of a virtual capacitance.

Again, the linear approach has given us a qualitative idea of what is going to happen. However for reasonable quantitative results, we again need to resort to numerically solving the non-linear differential equations (1),(2). Since the linear approach predicts that the absolute value of the virtual parallel reactance will increase with the gain factor of the feedback device necessary to sustain the oscillations, we expect the frequency deviation $\Delta\omega = \omega_{\text{osc}} - \omega_0$ to depend on the Q-factor of the LC tank and we shall therefore numerically investigate the deviation of the oscillation frequency for different Q-factors of the LC tank over a natural frequency range from 100kHz to 800kHz which is well inside the primary oscillation range. The results of this analysis have been plotted in figure 6.

As expected, the oscillation frequency ω_{osc} is below the natural frequency ω_0 of the LC tank and we see a pronounced dependency of the frequency deviation $\Delta\omega = \omega_{\text{osc}} - \omega_0$ on the Q-factor of the LC tank.

The plot in figure 6 has been created using a very high small signal, low frequency gain factor of the feedback device of $a_0 = 200000$. However, it turns out that the frequency deviation curve actually depends very little on a_0 and the curves for a_0 down to $a_0 = 5$ are practically identical to the curve shown. This behavior can be attributed to the fact that for high gain factors, the feedback device will be in saturation and hence inactive for most of the cycle, i.e. frequency skewing occurs only for a short time within each cycle. As the small signal gain factor decreases, the feedback device will have a smaller detuning effect but will be active (i.e. not in saturation) for a longer time within each cycle.

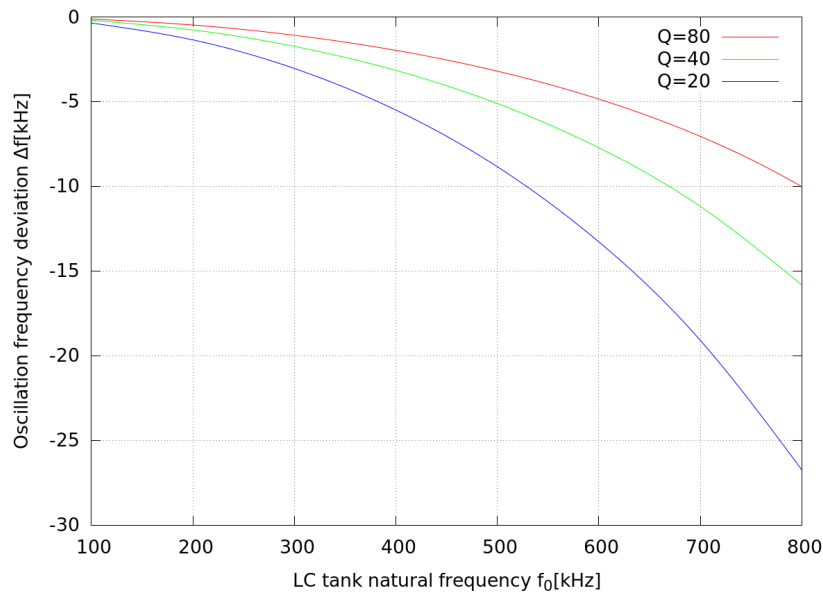


Figure 6: Frequency deviation

References

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- [4] K. Radhakrishnan and A. C. Hindmarsh, "Description and Use of LSODE, the Livermore Solver for Ordinary Differential Equations," LLNL report UCRL-ID-113855, December 1993.