Signal Levels and Leak Radiation of Loop Antennas for AM Home Transmitters

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Abstract

A considerable number of vintage radio sets for the AM broadcast bands use a build-in antenna without providing jacks to connect external antennas. Testing, maintenance, repair and alignment of these receivers therefore requires the use of home AM transmitters providing an amplitude modulated RF signal to the receiver. Since these build-in antennas are typically magnetic antennas (loop or ferrite rod), it is most convenient to also use a magnetic loop transmitting antenna and exploit local magnetic coupling to the receiver's build-in antenna. Besides creating the desired RF magnetic field for the receiver in question, there might be undesirable interferences with other nearby receivers. Also, the loop antenna will emit electromagnetic radiation, which for our purposes is unnecessary and therefore undesired since it further contributes to the electromagnetic pollution of the AM bands. The goal of this paper is to provide the reader with ready to use formulas to calculate the magnetic flux density of the local magnetic field and the equivalent signal level provided to the receiver as well as to estimate possible local interferences and the total power of undesired electromagnetic radiation emitted. It turns out that for reasonable signal levels provided to the receiver the danger of interference within apartment buildings or neighborhood areas can be practically eliminated by exploiting a steep $1/r^3$ decay in the magnetic field. Also, the total power of emitted electromagnetic radiation is typically in the picowatt range or below, making far-field interference neglectable for all practical purposes. Finally, it turns out that for reasonable signal levels, the power needed to drive the loop is typically in the lower microwatt range.

Acknowledgement

The Author wishes to thank Joseph L. Sousa for many valuable discussions on this topic. His ideas, remarks and always excellent questions from an engineering point of view greatly improved this paper.

This paper is organized as follows: First, general formulas to calculate the B-field (magnetic flux density) created by a rectangular loop at various spots and a formula for the electromagnetic radiation emitted by the loop are presented. In order to retain accessibility of this section to the more practically oriented reader, most of the calculations leading to these results have been moved to **appendix A**. Next comes the introduction of the equivalent signal level, relating the magnetic flux density created by the loop and provided to the receiver to a common measure of signal levels. These results are then applied to a large "room size" and a small "workbench size" loop to calculate the amplitude of the RF current in the loop necessary for reasonable signal levels to be provided to the receiver and the resulting near-field interference with other receivers as well as the electromagnetic radiation emitted by the loop. Finally, some considerations on providing well defined signal levels with RF voltage generators connected to loop transmitter antennas and required output power are presented. A summery of variables and constants used in the paper is given in **appendix B**. The reader may find it convenient to print this appendix and use it as a quick reference sheet when reading this paper.

Electrodynamics Of The Loop

By forcing an RF current $I(t)$ through a loop it can be used as a transmitter antenna. In this section, we will focus on it's general electrodynamic properties and derive ready to use formulas for our more practical considerations later on. Our considerations will be based on a rectangular loop in the xy-plane with the center at the origin of the 3-dimensional cartesian coordinate system and side lengths x_0 and y_0. This loop is depicted in figure 1

For convenience we will alternatively use spherical coordinates r, ϑ, φ. For these coordinates, we shall use the following convention generally adopted in physics where

$$\begin{aligned} x &= r\sin(\vartheta)\cos(\varphi) \\ y &= r\sin(\vartheta)\sin(\varphi) \\ z &= r\cos(\vartheta) \end{aligned}$$

and of course

$$r = \sqrt{x^2 + y^2 + z^2} = |\vec{r}|$$

In particular, ϑ is the angle between the imaginary line connecting a given point in space to the origin and the z-axis of the cartesian coordinate system.

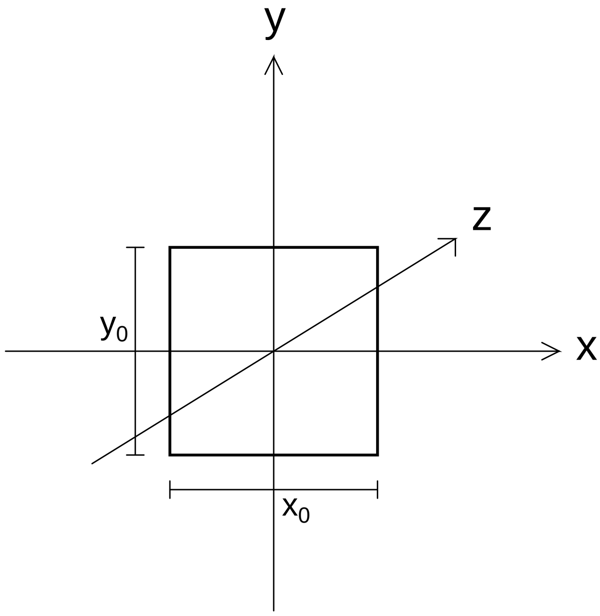

Figure 1: Position and orientation of the loop

Henceforth, we shall make the following assumptions about the loop. The size of the loop is small compared to the corresponding quarter wavelength $\lambda/4$ of the highest RF current frequency in the loop. The current $I(t)$ in the loop is therefore uniform. Furthermore, we assume that the ohmic resistance of the loop is neglectable, thus there is no voltage drop along the loop. Under these assumptions, closed-form expressions can be given for the electromagnetic fields generated by the loop for different points and regions in space. It should be noted that all obtained results can be transferred to a multi-turn loop provided that the distance (along the z-axis) between the individual turns is small compared to the size of the loop in the xy-plane and the overall length of the wire is still small compared to the corresponding quarter wavelength of the highest RF current frequency in the loop. In this case the current in the loop is still uniform and $I(t)$ can simply be replaced by $nI(t)$, where n is the number of turns.

The Magnetic Field At The Center Of The Loop

First of all, by virtue of the Biot–Savart law that can be obtained from Maxwell's equations, the magnetic flux density $\vec{B}$ (measured in Tesla) at the center of our loop can be calculated for a DC current I in the loop. The result is [3]

$$\vec{B}_C = \frac{2\mu_0 I}{\pi}\sqrt{\frac{1}{x_0^2} + \frac{1}{y_0^2}}\ \vec{n}$$

Where $\vec{n}$ is a unit vector perpendicular to the loop with it's direction determined by the "right-hand thumb rule" [4] and $\mu_0 = 12.56637... \cdot 10^{-7} \frac{\mathrm{Vs}}{\mathrm{Am}}$ is the magnetic constant. By simply replacing the DC current I with an alternating current

$$I(t) = \hat{I} \sin(\omega t)$$

and assuming a square loop with $x_0 = y_0 = l$ the absolute value of the B-field amplitude vector at the center of the loop is given by

$$\boxed{\hat{B}_C = |\hat{\vec{B}}_C| = \sqrt{8}\, \frac{\mu_0 \hat{I}}{\pi l}} \tag{1}$$

It should be noted that this so-called quasi-static approximation of simply replacing I with $I(t)$ is only possible if $\lambda/4$ of the RF current frequency ω is considerably bigger than the size of the loop. This, however, has already been one of our basic assumptions about our loop.

The Magnetic Near-Field Of The Loop

The next region in space we'll focus on is the so called near-field region where the distance r of the observer to the loop is considerably smaller than $\lambda/2\pi$ of the RF current frequency ω in the loop. We will restrict ourselves to the part of the near-field region where the distance r from the loop is significantly larger than the size of the loop, so that in combination our prerequisites for the formulas given in this section are $l \ll r \ll \lambda/2\pi$, where we have again assumed a square loop with side length l.

The magnetic flux density amplitude vector $\hat{\vec{B}}_N(\vec{r})$ of the near-field in the spherical coordinates introduced earlier is then given by

$$\hat{\vec{B}}_N(\vec{r}) = \frac{3\mu_0 a}{4\pi} \hat{I} \frac{1}{r^3} \begin{pmatrix} \sin(\vartheta)\cos(\varphi)\cos(\vartheta) \\ \sin(\vartheta)\sin(\varphi)\cos(\vartheta) \\ -\sin^2(\vartheta) + \frac{2}{3} \end{pmatrix}$$

The reader is referred to appendix A for a more detailed derivation from Maxwell's equations. From the above expression, the scalar amplitude $\hat{B}_N(\vec{r})$ of the magnetic flux density in the near field is

$$\boxed{\hat{B}_N(\vec{r}) = |\hat{\vec{B}}_N(\vec{r})| = \frac{3\mu_0 a}{4\pi} \hat{I} \frac{1}{r^3} \sqrt{\frac{4}{9} - \frac{1}{3}\sin^2(\vartheta)}} \tag{2}$$

It is worth noting that in this region the aspect ratio of the loop is no longer relevant. Only the area $a = x_0 y_0$ of the loop (respectively $a = l^2$ for the square loop) matters. In fact, it follows from appendix A that the shape of the loop no longer plays a role here. From the above equation it becomes obvious that the magnetic near-field has directional properties: It is the strongest along the z-axis ($\vartheta = 0$) and the weakest in the xy-plane ($\vartheta = 90°$). These directional properties are most easily visualized in a polar plot of magnetic flux density (absolute value) versus ϑ as shown in figure 2.

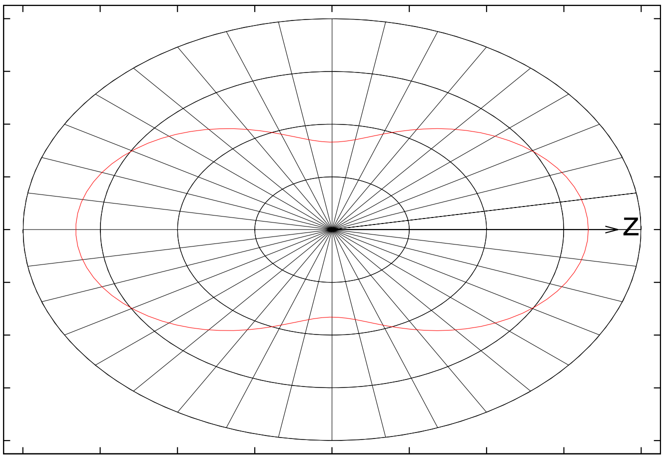

Figure 2: Directional properties of the magnetic near-field

Electromagnetic Far-Field And Radiation Of The Loop

The region in space where the distance r of the observer from the loop is significantly bigger than $\lambda/2\pi$ is called the far-field. An observer in the far-field region sees electromagnetic waves emanating from the loop and propagating outwards into space (for a detailed derivation see appendix A). These electromagnetic waves are approximately plane waves in this far-field region. We are not so much interested in the expressions for the E- and B-fields as we are in the expressions for the electromagnetic power being radiated by the loop (the reader interested in the fields is again referred to appendix A). After all, this is what is an undesired "side effect" of our attempts to use local magnetic coupling (induction) to feed a signal into our radio set. Our goal will always be to minimize the electromagnetic radiation "leaking out" from our loop antenna setup to prevent further pollution of the AM radio bands.

The electromagnetic radiation power density is given by the well known Poynting vector [5]. It is pointing in the direction of the energy flux, which is outwards away from the loop along imaginary lines connecting the observer to the origin where the loop is located. It's time averaged absolute value is the power density in watts per square meter for a small surface perpendicular to the direction of the Poynting vector. In the far-field of the loop the absolute value of the time averaged Poynting vector is given by (see appendix A)

$$S(\vec{r}) = \frac{\pi^2 \mu_0 a^2 \hat{I}^2 c}{2\lambda^4} \frac{1}{r^2} \sin^2(\vartheta) \tag{3}$$

Where c is the speed of light in a vacuum and also approximately in air. From the above expression, the far-field radiation pattern of the loop antenna can be visualized by a polar plot of the absolute value of the time averaged Poynting vector versus ϑ as shown in figure 3.

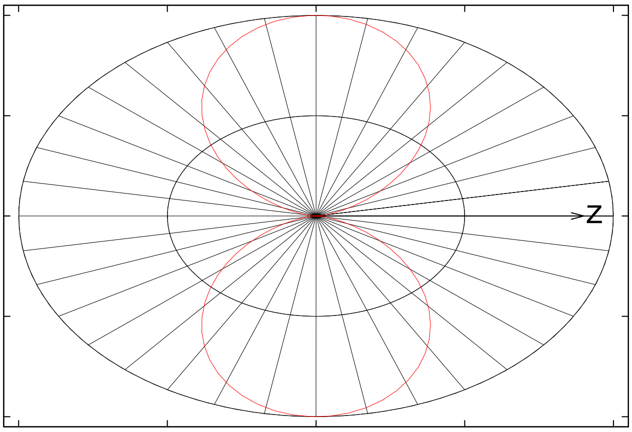

Figure 3: Far-field radiation pattern of the loop

Obviously, radiation reaches it's minimum (zero in the first order approximation used here) along the z-axis ($\vartheta = 0$) and has it's maximum in the xy-plane ($\vartheta = 90°$).

From the absolute value of the time averaged Poynting vector (3) the total power radiated by the loop can be obtained by integrating $S(\vec{r})$ over a closed surface in space. The result is (see appendix A)

$$\boxed{P = \frac{4}{3} \pi^3 \mu_0 \hat{I}^2 c \frac{a^2}{\lambda^4}} \tag{4}$$

Again, as in in the near-field region, the aspect ratio and even the shape of the loop is no longer relevant. Only the area $a = x_0 y_0$ (respectively $a = l^2$ for a square loop) of the loop matters.

Practical Applications Of The Obtained Results

Let us now focus on applying the formulas presented so far to practical examples of AM home transmitter loop antennas. We're in particular interested in providing reasonable signal levels to our own receivers while estimating possible near-field interference with other receivers as well as "leak radiation". It should be noted that the above formulas have been derived for free space, therefore the results they provide for interference and leak radiation present an upper boundary since our transmitter loop antenna will usually be indoors and other receivers will typically be behind walls or ceilings.

Local Magnetic Field And Signal Level

The formulas derived so far allow us to calculate the local magnetic field given as the magnetic flux density measured in Tesla at the spot were our radio receiver is located. This however is not really an intuitive figure and we need to find some way to relate this to more common measures for signal levels. Fortunately, this is quite simple. A signal level or signal strength in wireless communications is typically given as the scalar amplitude of the electric field strength $\hat{E}$ of an incoming plane electromagnetic wave in volts per meter. In our setup, we are however not exposing the radio receiver to an incoming plane electromagnetic wave from our home transmitter (for that we would have to place the receiver a couple of wavelengths away from the antenna which is not viable for the AM broadcast band frequencies). We are merely creating a local magnetic field to mimic the B-field of an incoming electromagnetic wave. For a plane electromagnetic wave in vacuum and approximately also in air the scalar E- and B-field amplitudes are related by [7]

$$\hat{E} = c\hat{B}$$

This relation is equivalent to the impedance of free space commonly used in electrical engineering [8]. We can now assign an equivalent scalar E-field amplitude to our locally generated B-field. It should again be stressed that this is not an actually generated electric field and has nothing to do with the electric near-field of the loop (see appendix A). We shall denote this equivalent scalar E-field amplitude by the letter σ to avoid confusion with the actual electric fields. Hence,

$$\boxed{\sigma = \hat{E}_{\text{equiv}} = c\hat{B}_{C/N}} \tag{5}$$

with the scalar amplitudes $\hat{B}_C$ respectively $\hat{B}_N$ of the locally generated B-fields from the previous section. We can now use σ as the equivalent signal level in volts per meter.

We now need to at least roughly get an idea of the signal levels that can be deemed "reasonable" when providing them to a typical vintage AM radio set. We will base our considerations on ITU recommendations for minimum signal levels from 1975 given in [9]. For the region containing Europe, minimum signal levels in the range from 63 dBμV/m (1413 μV/m) to 77 dBμV/m (7079 μV/m) are suggested. Since we are dealing with potential low-end receivers and noise in the AM bands has probably increased a lot since 1975, it seems in order to roughly double these figures. We therefore adopt a range from 3000 μV/m to 15000 μV/m as a "reasonable" signal level. In future years, these levels may have to be increased further since more and more applications, e.g. power line communication (PLC), create electromagnetic noise in the AM bands.

Big Loop With Receiver At The Center

Let us first look at a relatively big square loop with side a length of $l = 2\text{m}$ with the receiver located at the center of the loop. From equations (1) and (5) the equivalent signal level provided to the receiver is

$$\sigma = 3 \cdot 10^8 \frac{\text{m}}{\text{s}} \sqrt{8}\, \frac{12.56637 \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}}{\pi} \frac{\hat{I}}{2\text{m}} \approx 170 \frac{\text{V}}{\text{Am}} \cdot \hat{I}$$

Since our required signal level range is between 3000 μV/m and 15000 μV/m, the required current amplitude $\hat{I}$ of the RF current in the loop needs to be in the range between 17.6μA and 88μA.

With these results, the total radiated power of the big loop can be calculated. From equation (4) it follows that

$$P = \frac{4}{3}\pi^3 \cdot 12.56637 \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot 3 \cdot 10^8 \frac{\text{m}}{\text{s}} \left(\frac{l}{\lambda}\right)^4 \hat{I}^2 = 15.585 \cdot 10^3 \frac{V}{A} \left(\frac{l}{\lambda}\right)^4 \hat{I}^2$$

Since we're interested in the upper limit for the total radiated power, we'll use $\lambda = 200\text{m}$ (f =1.5Mhz). With $l = 2\text{m}$, the above equation turns out to be

$$P = 15.585 \cdot 10^{-5} \frac{\text{V}}{\text{A}} \cdot \hat{I}^2$$

For the RF current amplitude range of 17.6μA to 88μA, it turns out that the total radiated power does not exceed 1.2pW which is neglectable for all practical purposes.

We now need to estimate possible interference with other receivers in the near-field range of the big loop. This is done by determining the ratio $\hat{B}_C/\hat{B}_N$ of the magnetic flux density at the center of the loop to the magnetic flux density at a distance of $r = 5 \cdot l$ along the z-axis ($\vartheta = 0$) where the magnetic field is the strongest. Using equations (1) and (2) this becomes

$$\frac{\hat{B}_C}{\hat{B}_N} = \frac{\frac{\sqrt{8}\mu_0 \hat{I}}{\pi l}}{\frac{3\mu_0 l^2 \hat{I}}{4\pi(5l)^3}\sqrt{\frac{4}{9}}} = 250\sqrt{8} \approx 707.1$$

Obviously, at a distance of merely $r = 5 \cdot l$ the magnetic flux density has already dropped to roughly 1/700 of it's value at the center of the loop. By virtue of (5) this also applies to the equivalent signal levels. The reason for this steep decline is the $1/r^3$ dependence of the magnetic near-field as can be seen from equation (2). The conclusion here is, that as far as interference with other receivers is concerned the big loop can be considered "safe" if used in a single-family house. It can, however, not be deemed safe for apartment use since other receivers behind walls or ceilings might be too close to the loop.

Small Loop With Receiver In The Near-Field Region

We will now turn our attention to small loops with a side length of much less than 1 meter. With these loops, the receiver is typically located outside the loop, preferably along the z-axis ($\vartheta = 0$) since this is the region in space with the strongest B-field from the loop. Let's look at a small square loop with a side length of $l = 15$cm and the receiver located at a distance of $r = 1$m on the z-axis. Using these values in equation (2) we obtain by virtue of equation (5)

$$\sigma = 3 \cdot 10^8 \frac{\mathrm{m}}{\mathrm{s}} \frac{12.56637 \cdot 10^{-7} \frac{\mathrm{Vs}}{\mathrm{Am}}}{2\pi} \frac{(0.15\mathrm{m})^2}{(1\mathrm{m})^3} \cdot \hat{I} = 1.35 \frac{\mathrm{V}}{\mathrm{Am}} \cdot \hat{I}$$

for the equivalent signal level provided to the receiver. Again, our required signal level range is between $3000\,\mu\mathrm{V/m}$ and $15000\,\mu\mathrm{V/m}$. The RF current amplitudes $\hat{I}$ in the loop therefore need to be in the range from 2.2 mA to 11.1 mA.

With these results we can now calculate the total radiated power of this small loop. Again, from equation (4) it follows that

$$P = \frac{4}{3}\pi^3 \cdot 12.56637 \cdot 10^{-7} \frac{\mathrm{Vs}}{\mathrm{Am}} \cdot 3 \cdot 10^8 \frac{\mathrm{m}}{\mathrm{s}} \left(\frac{l}{\lambda}\right)^4 \hat{I}^2 = 15.585 \cdot 10^3 \frac{V}{A} \left(\frac{l}{\lambda}\right)^4 \hat{I}^2$$

Inserting $l = 0.15$m and $\lambda = 200$m then yields

$$P = 4.93 \cdot 10^{-9} \frac{\mathrm{V}}{\mathrm{A}} \cdot \hat{I}^2$$

From which it follows that the total radiated power for the above RF current amplitudes $\hat{I}$ does not exceed 0.61pW. This, again, is neglectable for all practical purposes.

Let's now estimate possible interference with other receivers in the near-field range of the small loop. As pointed out before, from equation (2) it immediately follows that the magnetic flux density decays over distance according to $1/r^3$. Therefore, by increasing the distance from 1m to 3m from the loop, the equivalent signal level will drop by a factor of 27. Increasing the distance to 5m, the signal level will be decreased by a factor of 125. The conclusion here is that the small loop, as far as interference goes, is relatively safe to use even in an apartment building, provided that the location of the loop is chosen to be reasonably far away from inter-apartment walls and ceilings.

Providing Well Defined Signal Levels To A Receiver

As has become clear by now, it is crucial to measure and adjust the amplitude of the RF current in the loop. This is most easily accomplished by connecting a resistor in series with the loop and measuring the RF voltage drop with a high impedance device. Typically, one would use an oscilloscope with a 10x or even 100x voltage divider probe to also keep the input capacitance as low as possible. An example schematic of such a setup is given in figure 4

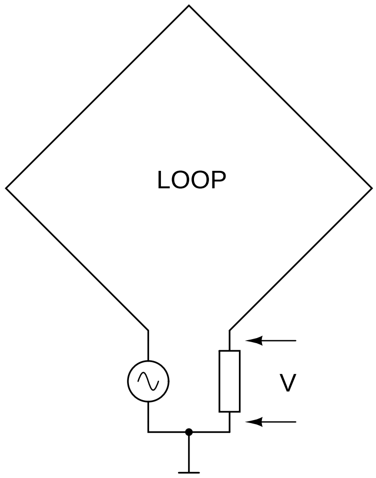

Figure 4: Setup to measure the RF current in the loop

It should be pointed out that if the loop antenna is driven by a voltage source, the RF current in the loop will not only depend on the voltage provided to the loop by the RF generator, but also on the properties of the receiver and the frequency the receiver is tuned to. This is all due to the fact that the magnetic coupling used here between the loop transmitter antenna and the receiver is always reciprocal. The RF current in the receiver's magnetic antenna (that typically serves as the inductor of it's input tuned circuit) will also induce a voltage into the loop transmitter antenna adding to the voltage of the RF generator. Hence, tuning the receiver will alter the RF current in the transmitter loop and therefore also the equivalent signal level provided to the receiver. This influence will be small for a loose coupling between the loop and the receiver. However, it might still be necessary to monitor the RF current in the transmitter loop and adjust the RF generator output accordingly while tuning the receiver. Bearing this in mind and using appropriate methods to measure the amplitude of the RF current in the transmitter loop, one is able to provide well defined signal levels to a receiver. This makes it possible to perform e.g. experiments on receiver sensitivity.

Another application is the very cost-efficient approximate measurement of noise levels in the AM bands. For this task, one would typically use an AM receiver with a high RF/IF amplification factor, controlled by an automatic gain control circuit. For general noise measurements, this receiver is tuned to an "empty" frequency so that only audio frequency noise detected from the RF noise being received can be heard from the speaker. For this purpose the receiver must not have an automatic mute feature or it must be turned off. The AM home transmitter is then switched on and the signal level is increased until it's audio signal starts to emerge from the noise floor and can be heard from the speaker. The equivalent signal level provided by the home AM transmitter at this point is then a rough measure of the RF noise at this particular frequency. In order to measure the noise level of a specific RF noise source (e.g. a nearby switched mode power supply unit) one would tune the receiver to this noise source and see what transmitter signal level it takes to overcome the hum or buzz that can be heard from this specific noise source on the receiver.

Estimating The Input Power Needed By The Loop

By now, we know the magnitude of the RF currents in the loop necessary to provide reasonable signal levels to a receiver. What we don't yet know is the output power a signal generator driving the loop needs to have. Fortunately, estimating the power necessary to drive the loop with the RF current amplitudes given in the previous sections is quite simple. The power provided by the signal generator is equal to the power dissipated in the loop. The power dissipation of the loop is the sum of the power radiated by the loop, the power drawn by the receiver due to magnetic coupling and the losses in the wire the loop is made of. Since the power radiated by the loop is typically in the picowatt range it can certainly be neglected. The power drawn by the receiver is typically in the lower nanowatt range (refer to appendix A) and can also be neglected against the losses in the loop wire. We are therefore left with the task of calculating the power loss in the loop wire.

If the inductance of the loop is compensated by a (lossless) capacitance and the resulting circuit is operated at the resonant frequency ω_0, the Q-factor of this tuned circuit and therefore the loop is [10]

$$Q = \frac{\omega_0 L}{R} \tag{6}$$

where L is the inductance of the loop and R is the equivalent (series) loss resistance. Since the loss resistance is ohmic, it's power dissipation P_R is

$$P_R = I_{\mathrm{R}MS}^2 \cdot R = \frac{1}{2}\hat{I}^2 \cdot R$$

since $I_{\mathrm{R}MS} = \frac{1}{\sqrt{2}}\hat{I}$. Solving (6) for R and using the result in the above equation yields:

$$P_R = \frac{1}{2}\hat{I}^2 \cdot \frac{\omega_0 L}{Q}$$

The inductance of the loop varies with it's size and the wire gauge used. To get a rough idea of the magnitude of P_d we'll use $L = 10\mu$H. The Q-factor is assumed to be $Q = 60$ for a resonant frequency of $f = 600$kHz. The power dissipated is then

$$P_R = \frac{1}{2}\hat{I}^2 \cdot \frac{2\pi \cdot 600 \cdot 10^3 \frac{1}{s} \cdot 10 \cdot 10^{-6}\mathrm{H}}{60} = \hat{I}^2 \cdot 0.31\frac{\mathrm{V}}{\mathrm{A}} \tag{7}$$

since the SI inductance unit Henry [H] can be given as [11]

$$1\mathrm{H} = 1\frac{\mathrm{Vs}}{\mathrm{A}}$$

Substituting the RF current amplitudes $\hat{I}$ from the previous sections in equation (7) we obtain a power dissipation of not more than 40μW. Hence, the signal generator driving the loop will typically have to provide output power in the lower microwatt range.

Appendix A: Derivation Of Previously Used Formulas

Maxwell's Equations

The behavior of electric and magnetic fields in a vacuum and in good approximation also in air is governed by the following partial differential equations [2]

$$\nabla \cdot \vec{E}(\vec{r},t) = \frac{1}{\epsilon_0}\rho(\vec{r},t)$$

$$\nabla \cdot \vec{B}(\vec{r},t) = 0$$

$$\nabla \times \vec{E}(\vec{r},t) = -\frac{\partial}{\partial t}\vec{B}(\vec{r},t)$$

$$\nabla \times \vec{B}(\vec{r},t) = \mu_0\vec{j}(\vec{r},t) + \mu_0\epsilon_0\frac{\partial}{\partial t}\vec{E}(\vec{r},t)$$

commonly known as Maxwell's equations. For an explanation of the variables used see appendix B. The speed of light c in a vacuum and in good approximation also in air used later on is determined by μ_0 and ϵ_0 according to

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

Maxwell's equations allow for the introduction of a scalar potential $\Phi(\vec{r},t)$ and a vector potential $\vec{A}(\vec{r},t)$ from which the electric field can be calculated as

$$\vec{E}(\vec{r},t) = -\frac{\partial}{\partial t}\vec{A}(\vec{r},t) - \nabla\Phi(\vec{r},t) \tag{8}$$

and the magnetic field is given by

$$\vec{B}(\vec{r},t) = \nabla \times \vec{A}(\vec{r},t) \tag{9}$$

It should be noted that $\Phi(\vec{r},t)$ and $\vec{A}(\vec{r},t)$ can be altered in such ways that the electric and magnetic fields calculated from them do not change. These alterations are called gauge transformations. By choosing the Lorentz gauge transformation given by

$$\nabla \cdot \vec{A}(\vec{r},t) + \frac{1}{c^2}\frac{\partial}{\partial t}\Phi(\vec{r},t) = 0$$

we obtain two uncoupled inhomogeneous wave equations for the scalar and vector potential of the form

$$\Delta\Phi(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Phi(\vec{r},t) = -\frac{1}{\epsilon_0}\rho(\vec{r},t)$$

$$\Delta\vec{A}(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{A}(\vec{r},t) = -\mu_0\vec{j}(\vec{r},t)$$

The general solution of the above wave equations is given by the volume integrals

$$\Phi(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int dV' \; \frac{\rho(\vec{r'},t-\frac{1}{c}|\vec{r}-\vec{r'}|)}{|\vec{r}-\vec{r'}|} \tag{10}$$

and

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int dV' \; \frac{\vec{j}(\vec{r'},t-\frac{1}{c}|\vec{r}-\vec{r'}|)}{|\vec{r}-\vec{r'}|} \tag{11}$$

A detailed explanation and rigorous derivation of all this can be found in most text books on electrodynamics, e.g. in [1].

First Order Approximation Of The Vector Potential

In general, there will be no closed form expression for the volume integrals above. However, our chances of finding a closed-form solution will rise if we are dealing with a charge density $\rho(\vec{r},t)$ and a current density $\vec{j}(\vec{r},t)$ limited to a small region in space and are looking at points $\vec{r}$ sufficiently far away from this region. For convenience we choose the origin of our coordinate system to be within this small region. We then have $|\vec{r'}| \ll |\vec{r}|$ and are able to do a series expansion up to the first order of the integrands in the above volume integrals. This results in [1]

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\frac{1}{r}\vec{A}_1(\vec{r},t) + \frac{\mu_0}{4\pi}\left(\frac{1}{r^3} + \frac{1}{cr^2}\frac{\partial}{\partial t}\right)\vec{A}_2(\vec{r},t) \tag{12}$$

where

$$\vec{A}_1(\vec{r},t) = \int dV' \; \vec{j}(\vec{r'},t-r/c) \tag{13}$$

and

$$\vec{A}_2(\vec{r},t) = \int dV' \; (\vec{r}\cdot\vec{r'})\,\vec{j}(\vec{r'},t-r/c) \tag{14}$$

with $r = |\vec{r}|$ denoting the distance from the origin of or coordinate system and therefore the distance from the aforementioned small region in space with a non-vanishing charge and current density.

The Vector Potential Of The Loop In First Order Approximation

We'll now apply the above formulas to the loop antenna described earlier. Since we assume the wire of the loop to be very thin, i.e. the diameter of the wire is very small compared to the dimensions of the loop we can approximate the current density with Dirac delta functions $\delta(x-x_0)$ having the well known properties

$$\delta(x-x_0) = 0 \quad ; x \neq x_0$$

and

$$\int_{-\infty}^{+\infty} \delta(x-x_0)dx = 1$$

The position vector $\vec{r}$ in this cartesian coordinate system can be expressed as

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

with it's length given by

$$|\vec{r}| = r = \sqrt{x^2+y^2+z^2}$$

Similar, the current density vector in cartesian coordinates is

$$\vec{j}(\vec{r},t) = \begin{pmatrix} j_x(x,y,z,t) \\ j_y(x,y,z,t) \\ j_z(x,y,z,t) \end{pmatrix}$$

Since the total length of the wire making up the loop is assumed to be considerably smaller than $\lambda/4$ there is an approximately uniform current $I(t)$ in the loop. The components of the current density vector are therefore

$$\begin{aligned}
j_x(x,y,z,t) &= \begin{cases} I(t)\delta(z)\left(\delta(y+\frac{1}{2}y_0) - \delta(y-\frac{1}{2}y_0)\right) & ; -\frac{x_0}{2} \leq x \leq \frac{x_0}{2} \\ 0 & ; \text{else} \end{cases} \\
j_y(x,y,z,t) &= \begin{cases} I(t)\delta(z)\left(\delta(x-\frac{1}{2}x_0) - \delta(x+\frac{1}{2}x_0)\right) & ; -\frac{y_0}{2} < y < \frac{y_0}{2} \\ 0 & ; \text{else} \end{cases} \\
j_z(x,y,z,t) &= 0
\end{aligned}$$

Substituting x, y, z, t with $x', y', z', t - r/c$ in (13) and (14) and performing the integration then yields

$$\vec{A}_1(\vec{r},t) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

and

$$\vec{A}_2(\vec{r},t) = I(t - r/c)\, x_0 y_0 \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$$

Using these results in (12) gives

$$\vec{A}(\vec{r},t) = \frac{\mu_0 a}{4\pi} \left(\frac{1}{r^3} + \frac{1}{cr^2} \frac{\partial}{\partial t} \right) I(t - r/c) \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} \tag{15}$$

for the vector potential $\vec{A}(\vec{r},t)$ for positions sufficiently far away from the loop. In the above equation the area covered by the loop has been denoted by $a = x_0 y_0$. It should be noted that a loop consisting of several turns of wire close together on the same frame can also be described by the above vector potential if the loop current I is replaced by the product of the number of turns n and the current in the wire I. In this case, it is the total length of the wire that must be considerably smaller than $\lambda/4$ to have an approximately uniform current in the loop.

It it worth mentioning that the previous approximation where we assume the loop to be limited to a relatively small region in space and look at positions sufficiently far away from the loop has one immediate effect: The shape of the loop does not affect the vector potential and hence the E- and B-fields. Only the area a covered by the loop matters.

Starting with the above expression we can develop further approximations valid for $r \ll \lambda/2\pi$ respectively $r \gg \lambda/2\pi$. These approximations are called near-field respectively far-field approximation. We first look at a periodic function $I(t)$ of the form

$$I(t) = \hat{I} \sin(\omega t)$$

and substitute this into the above expression. Bearing in mind that $\omega/c = 2\pi/\lambda$ the result is:

$$\vec{A}(\vec{r},t) = \frac{\mu_0 a}{4\pi} \hat{I} \left(\frac{1}{r^3} \sin(\omega(t - r/c)) + \frac{2\pi}{\lambda r^2} \cos(\omega(t - r/c)) \right) \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$$

In the above sum, the first term is generated by the $1/r^3$ factor in equation (15) while the second term is generated by the differential operator. In case $r \ll \lambda/2\pi$

the second term of the above sum can be neglected against the first term and therefore the differential operator in (15) can be dropped. Furthermore, in this case $I(t - r/c) = I(t - rT/\lambda)$ can be approximated by $I(t)$.

If however $r \gg \lambda/2\pi$, the first term in the above sum can be neglected against the second one and therefore the $1/r^3$ factor in (15) can be dropped. Also, $I(t-r/c)$ stays as it is.

These considerations do not only apply to sinusoidal loop currents $I(t)$ but by virtue of Fourier's theorem to any periodic loop current, provided that all physically relevant harmonics satisfy the aforementioned conditions.

The Near-Field Approximation

We will first look at the region where $r \ll \lambda/2\pi$. From the above considerations it follows that the near-field vector potential $\vec{A}_N$ can be written as

$$\vec{A}_N(\vec{r},t) = \frac{\mu_0 a}{4\pi}\frac{1}{r^3}I(t)\begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} \tag{16}$$

The magnetic flux density $\vec{B}_N(\vec{r},t)$ of the near-field is given by (9). The calculation of the partial derivatives of the components of $\vec{A}_N(\vec{r},t)$ is straight forward, finally yielding

$$\vec{B}_N(\vec{r},t) = \frac{3\mu_0 a}{4\pi}I(t)\frac{1}{r^5}\begin{pmatrix} xz \\ yz \\ -(x^2+y^2) \end{pmatrix} + \frac{\mu_0 a}{2\pi}I(t)\frac{1}{r^3}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

At this point it is useful to switch the coordinate system to the spherical coordinates introduced earlier. Furthermore, we assume that the current $I(t)$ in the loop is given by

$$I(t) = \hat{I}\sin(\omega t)$$

After switching to spherical coordinates, the magnetic flux density $\vec{B}_N(\vec{r},t)$ of the near-field can be expressed as:

$$\vec{B}_N(\vec{r},t) = \frac{3\mu_0 a}{4\pi}\hat{I}\sin(\omega t)\frac{1}{r^3}\begin{pmatrix} \sin(\vartheta)\cos(\varphi)\cos(\vartheta) \\ \sin(\vartheta)\sin(\varphi)\cos(\vartheta) \\ -\sin^2(\vartheta)+\frac{2}{3} \end{pmatrix}$$

For the sake of completeness we shall also calculate the electric near-field although it is not needed for our purposes here. The E-field is calculated from the scalar potential $\Phi(\vec{r},t)$ and the vector potential $\vec{A}(\vec{r},t)$ according to equation (8). Since we have made the assumption that the resistance of the loop is neglectable (hence there is no voltage drop along the loop) and the circumference of the loop is much smaller than $\lambda/4$ (hence the current is uniform) the loop is neutral at every point. Therefore, $\rho(\vec{r}, t) = 0$ and it follows from (10) that

$$\Phi(\vec{r}, t) = 0$$

Equation (8) therefore simplifies to

$$\vec{E}(\vec{r}, t) = -\frac{\partial}{\partial t}\vec{A}(\vec{r}, t)$$

Substituting $\vec{A}_N(\vec{r}, t)$ from (16) then yields

$$\vec{E}_N(\vec{r}, t) = \frac{\mu_0 a}{4\pi}\frac{1}{r^3}\frac{\partial I(t)}{\partial t}\begin{pmatrix} y \\ -x \\ 0 \end{pmatrix}$$

Using

$$I(t) = \hat{I}\sin(\omega t)$$

in the above equation and switching again to spherical coordinates we finally obtain

$$\vec{E}_N(\vec{r}, t) = \frac{\mu_0 a \omega \hat{I}}{4\pi}\frac{1}{r^2}\cos(\omega t)\begin{pmatrix} \sin(\vartheta)\sin(\varphi) \\ -\sin(\vartheta)\cos(\varphi) \\ 0 \end{pmatrix}$$

The Far-Field Approximation

We will now turn our attention to the region where $r \gg \lambda/2\pi$. Applying our previous considerations to equation (15) we obtain

$$\vec{A}_F(\vec{r}, t) = \frac{\mu_0 a}{4\pi}\frac{1}{cr^2}\frac{\partial}{\partial t}I(t - r/c)\begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} \tag{17}$$

for the far-field vector potential $\vec{A}_F(\vec{r}, t)$ of the loop. Again, the E-field is calculated from the scalar potential $\Phi(\vec{r}, t)$ and the vector potential $\vec{A}(\vec{r}, t)$ according to equation (8). As in the near-field, we again have

$$\Phi(\vec{r}, t) = 0$$

and therefore equation (8) simplifies to

$$\vec{E}(\vec{r}, t) = -\frac{\partial}{\partial t}\vec{A}(\vec{r}, t)$$

Substituting $\vec{A}_F(\vec{r},t)$ from (17) then leads to

$$\vec{E}_F(\vec{r},t) = -\frac{\mu_0 a}{4\pi}\frac{1}{cr^2}\frac{\partial^2}{\partial t^2}I(t-r/c)\begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$$

If we assume the loop current to be sinusoidal, hence

$$I(t) = \hat{I}\sin(\omega t)$$

this yields

$$\vec{E}_F(\vec{r},t) = \frac{\mu_0 a \hat{I}\omega^2}{4\pi c}\frac{1}{r^2}\sin(\omega(t-r/c))\begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} \tag{18}$$

Switching again to the spherical coordinates introduced earlier, equation (18) then turns into

$$\vec{E}_F(\vec{r},t) = \frac{\mu_0 a \hat{I}\omega^2}{4\pi c}\frac{1}{r}\sin(\omega(t-r/c))\begin{pmatrix} -\sin(\vartheta)\sin(\varphi) \\ \sin(\vartheta)\cos(\varphi) \\ 0 \end{pmatrix}$$

From the above expression, the amplitude vector $\hat{\vec{E}}_F(\vec{r})$ of the electric far-field is immediately recognized to be

$$\hat{\vec{E}}_F(\vec{r}) = \frac{\mu_0 a \hat{I}\omega^2}{4\pi c}\frac{1}{r}\begin{pmatrix} -\sin(\vartheta)\sin(\varphi) \\ \sin(\vartheta)\cos(\varphi) \\ 0 \end{pmatrix}$$

and hence, the scalar amplitude $\hat{E}_F(\vec{r})$ of the electric far-field can be written as

$$\hat{E}_F(\vec{r}) = |\hat{\vec{E}}_F(\vec{r})| = \frac{\mu_0 a \hat{I}\omega^2}{4\pi c}\frac{1}{r}|\sin(\vartheta)|$$

Calculating the magnetic flux density $\vec{B}_F(\vec{r},t)$ from the vector potential $\vec{A}_F(\vec{r},t)$ according to (9) is a little more tedious but poses no obstacles. The partial derivatives of the components of $\vec{A}_F(\vec{r},t)$ turn out to be

$$\frac{\partial A_{Fz}}{\partial y} = \frac{\partial A_{Fz}}{\partial x} = 0$$

and

$$\begin{aligned}
\frac{\partial A_{Fy}}{\partial z} &= \frac{\mu_0 a}{4\pi c} x\,\gamma \\
\frac{\partial A_{Fx}}{\partial z} &= -\frac{\mu_0 a}{4\pi c} y\,\gamma \\
\frac{\partial A_{Fy}}{\partial x} &= \frac{\mu_0 a}{4\pi c}\left(x\,\alpha + \frac{1}{r^2}\frac{\partial}{\partial t} I(t-r/c)\right) \\
\frac{\partial A_{Fx}}{\partial y} &= -\frac{\mu_0 a}{4\pi c}\left(y\,\beta + \frac{1}{r^2}\frac{\partial}{\partial t} I(t-r/c)\right)
\end{aligned}$$

with

$$\begin{aligned}
\alpha &= \frac{\partial}{\partial x}\left(\frac{1}{r^2}\frac{\partial}{\partial t} I(t-r/c)\right) \\
\beta &= \frac{\partial}{\partial y}\left(\frac{1}{r^2}\frac{\partial}{\partial t} I(t-r/c)\right) \\
\gamma &= \frac{\partial}{\partial z}\left(\frac{1}{r^2}\frac{\partial}{\partial t} I(t-r/c)\right)
\end{aligned}$$

Using $I(t) = \hat{I}\sin(\omega t)$ respectively $I(t-r/c) = \hat{I}\sin(\omega(t-r/c))$ we obtain

$$\alpha = \hat{I}\omega\left(\frac{-2x}{r^4}\cos(\omega(t-r/c)) + \frac{\omega}{c}\frac{x}{r^3}\sin(\omega(t-r/c))\right)$$

Since $r \gg \lambda/2\pi$ this can be approximated by

$$\alpha = \frac{\hat{I}\omega^2}{c}\frac{x}{r^3}\sin(\omega(t-r/c))$$

Calculating β and γ is done in the same way, yielding

$$\beta = \frac{\hat{I}\omega^2}{c}\frac{y}{r^3}\sin(\omega(t-r/c))$$

and

$$\gamma = \frac{\hat{I}\omega^2}{c}\frac{z}{r^3}\sin(\omega(t-r/c))$$

This results in the following expression for the magnetic flux density $\vec{B}_F(\vec{r},t)$ of the far field of the loop:

$$\vec{B}_F(\vec{r},t) = \begin{pmatrix} -\frac{\mu_0 a \hat{I}\omega^2}{4\pi c^2}\frac{1}{r^3}\sin(\omega(t-r/c))\,xz \\ -\frac{\mu_0 a \hat{I}\omega^2}{4\pi c^2}\frac{1}{r^3}\sin(\omega(t-r/c))\,yz \\ \frac{\mu_0 a \hat{I}\omega}{4\pi c}\left(\frac{\omega}{c}\frac{1}{r^3}\sin(\omega(t-r/c))\,(x^2+y^2) + \frac{2}{r^2}\cos(\omega(t-r/c))\right) \end{pmatrix}$$

Switching to spherical coordinates this turns out to be

$$\vec{B}_F(\vec{r},t) = \frac{\mu_0 a\hat{I}\omega^2}{4\pi c^2}\frac{1}{r}\sin(\omega(t-r/c))\begin{pmatrix} -\sin(\vartheta)\cos(\vartheta)\cos(\varphi) \\ -\sin(\vartheta)\cos(\vartheta)\sin(\varphi) \\ \sin^2(\vartheta) \end{pmatrix}$$

where we have again neglected terms exhibiting a $1/r^2$ dependence against terms exhibiting a $1/r$ dependence. From the above equation it is easy to see that the amplitude vector $\hat{\vec{B}}_F(\vec{r})$ of the magnetic far-field has the form

$$\hat{\vec{B}}_F(\vec{r}) = \frac{\mu_0 a\hat{I}\omega^2}{4\pi c^2}\frac{1}{r}\begin{pmatrix} -\sin(\vartheta)\cos(\vartheta)\cos(\varphi) \\ -\sin(\vartheta)\cos(\vartheta)\sin(\varphi) \\ \sin^2(\vartheta) \end{pmatrix}$$

and the scalar amplitude $\hat{B}_F(\vec{r})$ of the magnetic far-field is

$$\hat{B}_F(\vec{r}) = \frac{\mu_0 a\hat{I}\omega^2}{4\pi c^2}\frac{1}{r}|\sin(\vartheta)|$$

Comparing the expressions for the amplitude vectors of the E- and B-far-fields it becomes obvious that the scalar product $\hat{\vec{E}}_F(\vec{r})\cdot\hat{\vec{B}}_F(\vec{r})$ vanishes for all $\vec{r}$. The amplitude vectors of the E- and B-fields are therefore perpendicular to each other. Also, the scalar product of the position vector $\vec{r}$ with the E-field vector and the B-field vector vanishes. Therefore they are both perpendicular to an imaginary line connecting the observer at point $\vec{r}$ to the origin where the loop is located. Furthermore, from comparing the scalar amplitudes, it follows that

$$\hat{E}_F(\vec{r}) = c\hat{B}_F(\vec{r})$$

From those three properties of the amplitude vectors respectively scalar amplitudes it follows that an observer in the far-field region sees incoming plane electromagnetic waves emanating from the loop. We are mostly interested in the electromagnetic power being radiated by the loop. The radiation power density is given by the Poynting vector [5]

$$\vec{S}(\vec{r},t) = \frac{1}{\mu_0}\left(\vec{E}(\vec{r},t)\times\vec{B}(\vec{r},t)\right)$$

From this and the considerations above, it follows that the Poynting vector in the far-field always points outwards (away from the origin where the loop is located) along an imaginary line connecting the observer to the origin. Now that the direction of the Poynting vector is clear, we can focus on it's scalar value. For the plane electromagnetic waves that we have here in the far-field the time averaged scalar value $S(\vec{r})$ is [5]

$$S(\vec{r}) = \frac{\epsilon_0 c}{2}\hat{E}_F^2(\vec{r})$$

By substituting the expression for $\hat{E}_F(\vec{r})$ into the above equation and after some simplifications we arrive at

$$S(\vec{r}) = \frac{\pi^2 \mu_0 a^2 \hat{I}^2 c}{2\lambda^4} \frac{1}{r^2} \sin^2(\vartheta)$$

From this, the total radiated power of the loop P can be calculated. This is simply done by integrating the flux of the time averaged Poynting vector through an arbitrary closed surface containing the loop. Of course, the surface must be entirely in the far-field region. For convenience, we choose the surface to be a sphere with radius r and center at the origin. We have seen earlier that the Poynting vector points outwards along an imaginary line connecting the observer at point $\vec{r}$ to the origin. Hence, it is always parallel to the normal vector of the surface and the flux integral then becomes

$$P = \int_S S(\vec{r}) dO$$

Here, dO denotes the scalar surface element. We have chosen a different letter since S is already taken. In our spherical coordinate system, dO is given by

$$dO = r^2 \sin(\vartheta) d\vartheta d\varphi$$

and by also substituting $S(\vec{r})$ the above integral becomes

$$P = \frac{\pi^2 \mu_0 a^2 \hat{I}^2 c}{2\lambda^4} \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} \sin^3(\vartheta) d\vartheta d\varphi$$

Note that r cancels as expected. After performing the integration we end up with the following fairly simple expression for the total radiated power of the loop:

$$P = \frac{4}{3} \pi^3 \mu_0 \hat{I}^2 c \frac{a^2}{\lambda^4}$$

Power Dissipation In Lossy Tuned Circuits

A topic not directly related to the electrodynamics of the loop transmitter presented above is the behavior of the lossy tuned circuit. However, we need to get an idea of the magnitude of the power dissipated in the first tuned circuit of a receiver before the first amplification stage since this is the power drawn by the receiver from the magnetic near-field of the loop.

We turn our attention to the equivalent circuit of a lossy tuned circuit depicted in figure 5. The losses in the inductor and the capacitor are represented by a series ohmic loss resistance R. The power dissipated in this loss resistance is given by

$$P_{\text{loss}} = I_{\text{RMS}}^2 \cdot R \tag{19}$$

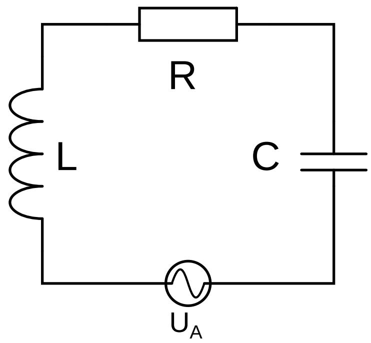

Figure 5: Equivalent of a lossy tuned circuit

The voltage U_{RMS} at the capacitor (which is the input voltage of the first amplification stage in typical receivers) is related to the current I_{RMS} in the circuit by

$$I_{\mathrm{RMS}} = \frac{U_{\mathrm{RMS}}}{|X_C|}$$

where $|X_C|$ denotes the absolute value of the reactance of the capacitor. Since $|X_C| = 1/\omega C$ the above equation turns into

$$I_{\mathrm{RMS}} = \frac{U_{\mathrm{RMS}}}{\frac{1}{\omega C}} = U_{\mathrm{RMS}} \cdot \omega C \tag{20}$$

The series loss resistance R is related to the Q-factor of the tuned circuit by [10]

$$Q = \frac{\omega L}{R}$$

Solving the above equation for R we get

$$R = \frac{\omega L}{Q} \tag{21}$$

Using equations (19), (20) and (21) we can calculate the power dissipation in the lossy tuned circuit. Note, that it is more intuitive to calculate R and I separately than combining the above equations into one.

As a typical example, we assume the following parameters: $L = 300\mu\mathrm{H}$, $C = 235\mathrm{pF}$, hence the resonant frequency is approximately $f = 600\mathrm{kHz}$. The Q-factor is assumed to be $Q = 30$. If there is a convenient $2\mathrm{V}_{\mathrm{RMS}}$ after RF/IF amplification available for demodulation and the RF/IF amplification factor is 100, the input voltage at the first stage is $20\mathrm{mV}_{\mathrm{RMS}}$. This is the voltage U_{RMS} at the capacitor. We therefore get from (20)

$$I_{\mathrm{RMS}} = 20\mathrm{mV} \cdot 2\pi \cdot 600\mathrm{kHz} \cdot 235\mathrm{pF} \approx 18\mu\mathrm{A}$$

and from (21) we get

$$R = \frac{2\pi \cdot 600\mathrm{kHz} \cdot 300\mu\mathrm{H}}{30} \approx 38\Omega$$

Using these results in equation (19) we finally end up with

$$P_{\mathrm{loss}} \approx (18\mu\mathrm{A})^2 \cdot 38\Omega \approx 12\mathrm{nW}$$

Appendix B: Variables And Constants

A note on the terminology of magnetic fields: There is a lot of confusion regarding the terms used to describe the "magnetic field" and it's "strength". Historically, the term "magnetic field" is used for the H-field, with H and B in a vacuum being related by $B = \mu_0 H$. The term "magnetic flux density" is commonly used to refer to the B-field. Informally and also in some more recent textbooks, the term 'magnetic field' is also used to refer to the B-field [6]. Since in this paper we're solely using the B-field, we shall also use the terms "magnetic field" and "magnetic flux density" interchangeably. The magnetic flux density B is measured in Tesla [T], where

$$1\,\mathrm{T} = 1\,\frac{\mathrm{Vs}}{\mathrm{m}^2}$$

in SI-units which are used throughout this paper.

List of selected variables and constants:

ω: Angular frequency ($\omega = 2\pi f$)

ω_0: Resonant angular frequency of a tuned circuit ($\omega_0 = 2\pi f_0$)

μ_0: Magnetic constant, $\mu_0 = 12.56637... \cdot 10^{-7}\,\frac{\mathrm{Vs}}{\mathrm{Am}}$

l: Side length of the loop whenever assumed to be square

a: Area of the loop

c: Speed of light in vacuum and approximately also in air, $c \approx 3 \cdot 10^8\,\frac{\mathrm{m}}{\mathrm{s}}$

λ: Wavelength, $\lambda = \frac{c}{f} = \frac{2\pi c}{\omega}$

$\hat{I}$: Amplitude of current $I(t)$

$I_{\mathrm{R}MS}$: Root mean square of current $I(t)$

R: Resistance (ohmic)

L: Inductance

ϵ_0: Electric constant, $\epsilon_0 = 8.854187... \cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}}$

ρ: Electric charge density

$\vec{j}$: Electric current density

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