Receiving Antennas For The AM Bands - Loop vs. Vertical Wire

Dipl.-Phys. Jochen Bauer

01/20/2013

Abstract

The two most common antenna types for AM reception are the loop antenna and the vertical wire antenna both having dimensions much smaller than the wavelengths of the electromagnetic waves in question. While the loop is a typical indoor antenna, the vertical wire antenna is by virtue of it's bigger size mostly an outdoor antenna. The performance of these two antenna types with respect to voltage pickup and power transfer is analyzed and compared. This is done for ideal, lossless setups as well as real, lossy setups of both antenna types. It is shown that even the theoretical upper limit for power harvesting from electromagnetic waves in the AM frequency range with lossless loop, respectively vertical wire antennas is much smaller than the power a pocket size solar panel could provide. For real, lossy antenna setups the different types of losses are analyzed and compared in magnitude. It is shown that the lossy vertical wire antenna outperforms the lossy loop antenna in typical setups when it comes to voltage pickup and power harvesting from an incoming electromagnetic wave.

This paper is organized as follows: First, the vertical wire antenna is analyzed. A formula for it's voltage pickup from an electromagnetic wave is presented and it's equivalent circuit with loss and radiation resistance is introduced. Next, the power that a receiver could draw from a lossless vertical wire antenna is calculated. Then, the different types of losses in the vertical wire antenna are described quantitatively, followed by calculations of the power that a typical lossy vertical wire antenna can provide to a receiver. In the second section, the multiturn loop antenna along with a formula for it's voltage pickup and equivalent circuit is introduced. The power a lossless loop antenna can provide to a receiver is calculated and compared to that of a vertical wire antenna. Next, the different types of losses in the loop antenna are analyzed quantitatively, followed by calculations of the power that a typical lossy loop antenna can provide to a receiver. Finally, the performance of both antennas is compared. A summery of variables and constants used in this paper is given in **appendix A**.

The Vertical Wire Antenna

We are looking at vertical wire antennas with a (physical) height h much smaller than a quarter wavelength ($\lambda/4$) of the incoming electromagnetic waves in question. One antenna terminal is the bottom end of the wire, while the other is the ground connection. A receiver with a finite input impedance is connected to the terminals, drawing an electric current $I(t)$ from the antenna. This setup is depicted in figure 1.

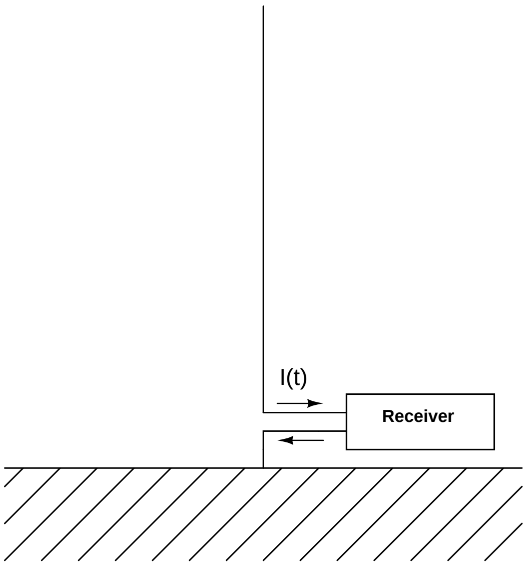

Figure 1: Vertical Wire Antenna

The current in the antenna must be equal to zero at the top end while it reaches it's maximum at the bottom end. The simplest assumption for the current distribution along the wire is therefore a linear distribution. This assumption turns out to be suitable for antennas much shorter than a quarter wavelength [1]. Based on the linear current distribution, the voltage distribution along the wire will therefore also be linear with the maximum at the top end and the minimum at the bottom end [1].

Voltage Pickup And Equivalent Circuit

The voltage picked up by the vertical wire antenna from an incoming electromagnetic wave and presented to the receiver is [1]

$$\boxed{\hat{U} = \frac{\hat{E} \cdot h}{2}} \tag{1}$$

where $\hat{U}$ denotes the amplitude of the voltage pickup $U(t)$ and $\hat{E}$ is the scalar amplitude of the electric field vector component of an incoming electromagnetic wave parallel to the wire. Note that if the antenna wire were a sufficiently short "ideal test-lead" with no inductance or capacitance connected to a receiver with an arbitrarily high input impedance (hence $I(t) = 0$ at every point of the antenna), the voltage pickup would be $\hat{U} = \hat{E} \cdot h$.

The vertical wire antenna, like any other antenna, can be described by an equivalent circuit where an ideal voltage source (the voltage pickup of the antenna) is connected in series with the antenna's reactance X_{A}, it's radiation resistance R_{rad} accounting for power loss due to electromagnetic radiation and it's loss resistance R_{loss}. The loss resistance in this case is composed of two parts: The loss resistance of the wire R_{w} and ground resistance R_{g}, the latter being due to the fact that the soil is not a perfect electric conductor. The resulting equivalent circuit for the vertical wire antenna is shown in figure 2

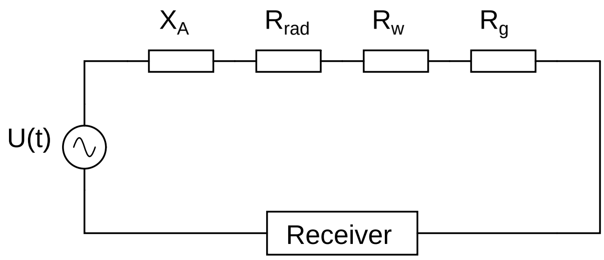

Figure 2: Vertical Wire Antenna Equivalent Circuit

For the reader not familiar with antenna theory, the concept of radiation resistance needs some further explanation: Any antenna can be used to receive **and** emit electromagnetic waves. This is commonly known as the property of reciprocity [2]. Hence, if a transmitter is connected to the terminals, giving rise to an RF current in the antenna, it will emit electromagnetic radiation. It turns out that the energy per time unit leaving the antenna, called the total radiated power P_{trp}, is proportional to I_{rms}^2, where I_{rms} is the root mean square of the RF current in the terminals. Since the power dissipated in an ohmic resistance R is given by $P = R \cdot I_{\mathrm{rms}}^2$ is seems natural to define the radiation resistance R_{rad} by

$$P_{\mathrm{trp}} = R_{\mathrm{rad}} \cdot I_{\mathrm{rms}}^2 \tag{2}$$

From the point of view of the transmitter, the radiation resistance acts like an ohmic resistor that dissipates power. However, the power "dissipated" in this resistor leaves the antenna in the form of electromagnetic radiation.

Ultimately, the cause for the emission of electromagnetic radiation by the antenna is the presence of an RF current in the antenna. It does not matter whether a transmitter, receiver or dummy load is connected to the terminals. Therefore, any antenna connected to a receiver (or dummy load) with finite input impedance allowing an RF current to be present in the antenna will loose a part of the power gathered from an incoming electromagnetic wave by becoming a radiator itself.

Power Transfer From The Lossless Vertical Wire Antenna

With the radiation resistance introduced above, we are ready to answer an important question: What is the maximum power transfer of an ideal (lossless) vertical wire antenna into a receiver? A lossless vertical wire antenna setup implies that the wire resistance R_w and the ground resistance R_g is zero, leaving us with the reactance X_A and the radiation resistance R_rad of the antenna. For maximum power transfer into the receiver, the reactance of the antenna X_A needs to be compensated. This can be accomplished by resonating with an equal and opposite reactance $-X_\mathrm{A}$ in the receiver input circuit. Full reactance compensation can in general only be achieved for a single frequency. For example, an antenna behaving mostly like a capacitance for a given frequency (like the short vertical wire antenna [1]) is connected in series with a variable inductor and tuned to resonance. Furthermore, the (ohmic) input impedance R_in of the receiver must be matched to the output impedance of the antenna which in this case is it's radiation resistance R_rad. This entire setup is summarized in the equivalent circuit shown in figure 3

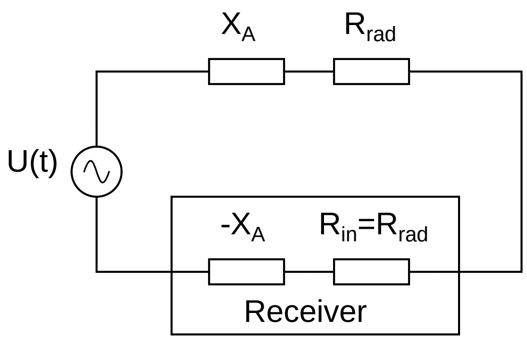

Figure 3: Ideal Vertical Wire Antenna

By virtue of Ohm's Law, the current I_rms in this circuit is given by

$$I_\mathrm{rms} = \frac{U_\mathrm{rms}}{R_\mathrm{rad} + R_\mathrm{in}}$$

and the power P_in dissipated in the receiver is

$$P_\mathrm{in} = I_\mathrm{rms}^2 \cdot R_\mathrm{in} = \left(\frac{U_\mathrm{rms}}{R_\mathrm{rad} + R_\mathrm{in}} \right)^2 \cdot R_\mathrm{in}$$

Since $R_{\text{in}} = R_{\text{rad}}$ and $\hat{U} = \sqrt{2} \cdot U_{\text{rms}}$ for sinusoidal voltage functions, where $\hat{U}$ is the amplitude of the voltage picked up by the antenna, we get

$$P_{\text{in}} = \frac{\hat{U}^2}{8R_{\text{rad}}} \tag{3}$$

By substituting the voltage pickup of the vertical wire antenna (1) into the above equation we finally obtain

$$P_{\text{in}} = \frac{\hat{E}^2 h^2}{32R_{\text{rad}}} \tag{4}$$

The radiation resistance R_{rad} of the vertical wire antenna can be calculated by using Maxwell's Equations to obtain an expression for the total radiated power and using the definition of the radiation resistance (2). In case of a vertical wire over a perfectly conducting surface, these calculations are greatly simplified by the fact that the radiation resistance of a monopole over a perfectly conducting surface is half the radiation resistance of it's equivalent dipole (the length of the equivalent dipole is twice the length of the monopole). The result is [6].

$$\boxed{R_{\text{rad}} = \frac{\pi}{3} Z_0 \left(\frac{h}{\lambda}\right)^2} \tag{5}$$

Where Z_0 is the impedance of free space [4]. Note that in [6] the approximation $Z_0 \approx 120\pi\Omega$ has been used. Substituting (5) into (4) yields

$$\boxed{P_{\text{in}} = \frac{\lambda^2 \hat{E}^2}{\frac{32}{3}\pi Z_0}} \tag{6}$$

This is the power P_{in} that a receiver that provides full reactance compensation and is fully impedance matched to the antenna can draw from an ideal, lossless vertical wire antenna over a perfectly conducting ground. It is remarkable that in this case P_{in} does not depend on the physical size of the antenna. This is specific to the lossless antenna setup. In actual (lossy) antenna setups, a bigger size of the antenna will typically improve it's performance.

In order to get figures for the maximum power a receiver can draw from an incoming electromagnetic wave, we need to substitute reasonable values in the above equation. We'll set $\lambda = 600\text{m}$ ($f = 500\text{kHz}$) and use electric field strengths $\hat{E}$ based on ITU recommendations for minimum signal levels for AM reception given in 1975 [7]. Specifically, we'll set $\hat{E} = 1500\mu\text{V/m}$ which can be considered a signal relatively weak but still sufficient for good reception in a low electromagnetic noise environment. Using these values we get $P_{\text{in}} = 64\mu\text{W}$. For comparison, if we look a a relatively strong signal of $\hat{E} = 15000\mu\text{V/m}$ the power transferred into the receiver is $P_{\text{in}} = 6.4\text{mW}$.

The reader is again reminded that the above figures are the theoretical upper limit for the power that can be drawn from an incoming electromagnetic wave by a lossless vertical wire antenna setup. The power transferred into the receiver by actual, lossy setups is considerably smaller. Still, we would like to get an idea of "how much power" $P_{\rm in} = 64\mu$W, respectively $P_{\rm in} = 6.4$mW is. This can best be done in the context of crystal radio sets. Crystal radio sets have the unique property of not relying on any external power sources. Power is drawn solely from the incoming electromagnetic wave the receiver is tuned to and a fraction of that power (depending on the efficiency of the design) is converted into sound power.

First of all, if one looks at the power a pocket size solar panel can provide to a regular receiver, which is typically somewhere between 0.1 Watts and 2 Watts, it becomes clear that a solar powered radio set can easily outperform any crystal radio set. In general, long-range radiative wireless energy transfer requires highly directional antennas at much higher frequencies. Current efforts in wireless mid-range energy transfer therefore rely on near-field coupling rather than emitting and receiving electromagnetic radiation [19]. On the other hand, if an ideal, lossless crystal set could convert $P_{\rm in} = 64\mu$W into sound power and output it over an earphone with a speaker area of 1cm^2 directly into the listener's ear, the resulting sound intensity [8] would be 0.64W/m^2 causing an acoustic sensation of "louder than a chainsaw at a distance of 1m" [9]. This example shows that there is practically unlimited headroom for improving the efficiency of crystal radio sets and the antennas connected to them.

Radiation Resistance And Loss Resistance

We will now turn our attention to the lossy vertical wire antenna. In addition to the radiation resistance, wire resistance $R_{\rm w}$ as well as ground resistance $R_{\rm g}$ will now play a role. Before we move on, we need to find out the magnitude of the radiation resistance $R_{\rm rad}$ for comparison with $R_{\rm w}$ and $R_{\rm g}$ later on. With the ground no longer being a perfect electric conductor, calculating the radiation resistance becomes quite difficult. In most cases one will have to resort to numerical methods and simulations. Fortunately, a very rough estimate of the radiation will suffice for comparison with $R_{\rm w}$ and $R_{\rm g}$ and hence we will calculate the radiation resistance under the assumption of a perfectly conducting ground. By substituting $\lambda = 600$m ($f = 500$kHz) into (5) and assuming a reasonable length of the vertical wire (physical height of the antenna) of $h = 5$m we get $R_{\rm rad} \approx 27$mΩ.

Wire Resistance

Next, we need to look at the (ohmic) resistance of the wire. For RF currents, the resistance of the wire is mostly governed by the skin effect [10] causing the current to flow mainly near the surface of the conductor, making the resistance R_w considerably larger than the DC resistance of the wire. The distance from the surface of the conductor where the current density has dropped to $1/e$ (≈ 0.37) is called the skin depth. For good conductors, the skin depth is approximately given as [10]

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}}$$

where ω is the angular frequency of the RF current, ρ is the resistivity and μ the absolute magnetic permeability of the material the conductor is made of. The physical resistance of a long, cylindrical wire of length L, diameter D and resistivity ρ can be approximated by [10]

$$R_\mathrm{w,phys} = \frac{L\rho}{\pi D\delta}$$

provided that the skin depth δ is much smaller than the diameter D of the wire. Combining the two expressions above and using $\omega = 2\pi c/\lambda$ as well as $D = 2r_\mathrm{w}$ then yields

$$\boxed{R_\mathrm{w,phys} = \frac{L}{r_\mathrm{w}}\lambda^{-\frac{1}{2}} \cdot \frac{1}{2\pi}(\pi c\mu\rho)^{\frac{1}{2}}} \tag{7}$$

If we focus on wires made of copper we can set $\mu = 1.26 \cdot 10^{-6}\,\mathrm{H/m}$ and $\rho = 1.68 \cdot 10^{-8}\,\Omega\mathrm{m}$ ([11],[12]) ending up with

$$R_\mathrm{w,phys} = 0.71 \cdot 10^{-3}\,\Omega\mathrm{m}^{\frac{1}{2}} \cdot \frac{L}{r_\mathrm{w}}\lambda^{-\frac{1}{2}} \tag{8}$$

Using the same antenna parameters as above ($L = h = 5\mathrm{m}$, $\lambda = 600\mathrm{m}$) and assuming a wire radius of $r_\mathrm{w} = 0.5\mathrm{mm}$ we arrive at a physical wire resistance of $R_\mathrm{w,phys} \approx 290\mathrm{m}\Omega$.

However, the physical resistance $R_\mathrm{w,phys}$ of the wire is not equal to the wire resistance R_w from the antenna's equivalent circuit (figure 2). This is because the current amplitude (resp. RMS of the current) in the antenna wire is not uniform along the wire, while in the equivalent circuit the current through R_w is set to be the current in the antenna's terminals. Fortunately R_w can be deduced from $R_\mathrm{w,phys}$ quite easily: Since the current distribution along the antenna wire can be assumed to be linear [1], the RMS of the current in the antenna wire at height y above the ground can be written as

$$I_{\text{rms}}(y) = I_{\text{rms}}(0) \cdot (1 - y/h)$$

where $I_{\text{rms}}(0)$ is the RMS of the current in the antenna terminals. The physical resistance of the wire per unit length is $R_{\text{w,phys}}/h$, hence the resistance of an infinitesimally short wire segment of length dy is $(R_{\text{w,phys}}/h)dy$. Therefore the power dissipation in this infinitesimally short wire segment at position y is $dP_{\text{w}} = I_{\text{rms}}^2(y) \cdot (R_{\text{w,phys}}/h)dy$. The total power dissipated in the antenna wire is then

$$P_{\text{w}} = \int_0^h I_{\text{rms}}^2(y) \cdot (R_{\text{w,phys}}/h)dy$$

Using $I_{\text{rms}}(y)$ as given above and performing the integration yields

$$P_{\text{w}} = \frac{1}{3} I_{\text{rms}}^2(0) R_{\text{w,phys}}$$

Since the wire resistance R_{w} in the equivalent circuit is defined by $P_{\text{w}} = I_{\text{rms}}^2(0)R_{\text{w}}$ (bear in mind that $I_{\text{rms}}(0)$ is the RMS of the current in the antenna terminals), we see that

$$\boxed{R_{\text{w}} = \frac{1}{3} R_{\text{w,phys}}} \tag{9}$$

For our example above, the wire resistance is therefore $R_{\text{w}} = \frac{1}{3} \cdot 290\text{m}\Omega \approx 97\text{m}\Omega$

Ground Resistance

The earth is not a perfect electric conductor and any grounding system has a non-zero resistance R_{g} to earth since there can only be a finite contact surface between the grounding system in question and the surrounding soil. Theoretical and semi-empirical formulas for R_{g} for various grounding systems are given in [13]. A common grounding system used with vertical wire antennas is a metal rod driven into the ground known as a “driven rod”. A semi-empirical formula for the ground resistance of the driven rod is [13]

$$\boxed{R_{\text{g}} = \frac{\rho_{\text{g}}}{2\pi L_{\text{rod}}} \ln \frac{3L_{\text{rod}}}{D_{\text{rod}}}} \tag{10}$$

where ρ_{g} is the resistivity of the soil surrounding the rod, L_{rod} is the length of the rod in the ground and D_{rod} it’s diameter. A table of typical soil resistivities is also given in [13]. If we take the length of the rod in the ground to be $L_{\text{rod}} = 1\text{m}$ and it’s diameter to be $D_{\text{rod}} = 3\text{cm}$ the above formula yields

$$R_{\text{g}} = 0.73 \frac{1}{\text{m}} \cdot \rho_{\text{g}}$$

For moist soil with a resistivity of typically $\rho_{\text{g}} = 10^2\,\Omega\text{m}$ we get $R_{\text{g}} = 73\Omega$ while for dry soil with typically $\rho_{\text{g}} = 10^3\,\Omega\text{m}$ the ground resistance is $R_{\text{g}} = 730\Omega$.

It becomes obvious that the radiation resistance R_{rad} and the wire resistance R_{w} can be neglected against the ground resistance R_{g} for the vertical wire antenna. Any efforts to improve the performance of the vertical wire antenna by lowering it's loss resistance should therefore go into improving the ground connection.

Power Transfer From The Lossy Vertical Wire Antenna

With the loss resistance of the vertical wire antenna being dominated by it's ground resistance R_{g}, the power transfer into a receiver under full reactance compensation and optimal impedance matching conditions ($R_{\text{in}} = R_{\text{g}}$) is

$$\boxed{P_{\text{in}} = \frac{\hat{E}^2 h^2}{32 R_{\text{g}}}} \tag{11}$$

The above expression is immediately obtained by substituting R_{g} for R_{rad} in (4). Let the physical height of the vertical wire antenna again be $h = 5\text{m}$. For a relatively weak signal of $\hat{E} = 1500\mu\text{V/m}$ and a ground resistance of $R_{\text{g}} = 73\Omega$ (moist soil) the power transfer into the receiver is $P_{\text{in}} = 24\text{nW}$ while for a relatively strong signal of $\hat{E} = 15000\mu\text{V/m}$ it is $P_{\text{in}} = 2.4\mu\text{W}$. It becomes obvious that these figures are less than 1/1000 of the theoretical upper limit for the lossless vertical wire antenna.

The Loop Antenna

The loop antenna is the main alternative to the vertical wire antenna. Unlike the vertical wire antenna, it's considerably smaller size makes it a viable indoor antenna. We will look at a square, n-turn loop with side length s and a total wire length $L = 4sn$ much smaller than a quarter wavelength $\lambda/4$ of the electromagnetic waves in question. Hence, the current distribution along the wire is approximately uniform. A receiver with finite input impedance is connected to the terminals of the loop. This setup is depicted in figure 4.

Voltage Pickup And Equivalent Circuit

The voltage pickup of the n-turn Loop can easily be derived from Faraday's Law Of Induction [14]. The voltage induced in the loop is given as

$$U(t) = n\frac{d\Phi(t)}{dt}$$

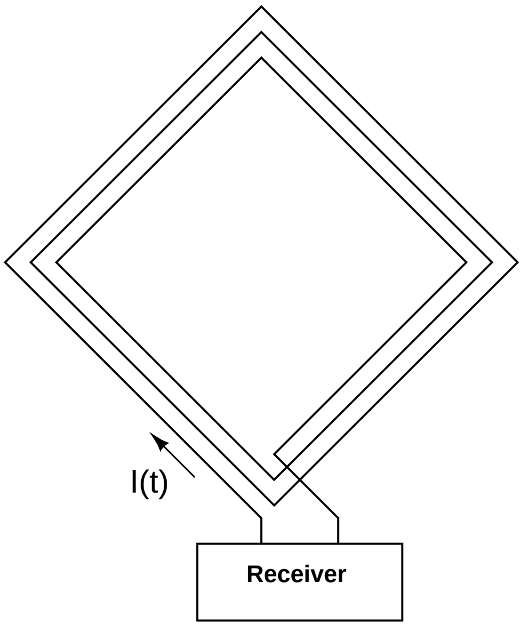

Figure 4: Multi-turn Loop Antenna

with the magnetic flux $\Phi(t)$ stemming from an incoming electromagnetic wave and being related to a magnetic flux density $B(t)$ at the location of the loop. Since the size of the loop is assumed to be much smaller than $\lambda/4$ of the incoming electromagnetic wave, $B(t)$ is approximately uniform over the area of the loop. With $\Phi(t) = A \cdot B(t)$, where A denotes the area of the loop we get

$$U(t) = nA\frac{dB(t)}{dt}$$

Using $B(t) = \hat{B}\sin(\omega t)$, where $\hat{B}$ is the absolute value of the B-field vector component perpendicular to the loop of an incoming electromagnetic wave, the above equation becomes

$$U(t) = nA\hat{B}\omega\cos(\omega t)$$

The amplitude $\hat{U}$ of the voltage pickup $U(t)$ is therefore

$$\hat{U} = nA\hat{B}\omega$$

In plane electromagnetic waves, the B-field (magnetic flux density) and E-field amplitudes are related by $E = cB$ with c being the speed of light in a vacuum and also approximately in air [15]. If the magnetic flux density B is replaced by the magnetic field strength $H = \frac{1}{\mu_0}B$, this relation becomes $E = c\mu_0 H = Z_0 H$, where $Z_0 = c\mu_0$ is commonly known in electrical engineering as the impedance of free space [4]. Using the initial form $E = cB$ along with $A = s^2$ for the square loop and $\lambda = 2\pi c/\omega$, the above equation yields:

$$\boxed{\hat{U} = \frac{2\pi s^2 n}{\lambda}\hat{E}} \tag{12}$$

where $\hat{E}$ is the absolute value of the E-field vector component parallel to the loop of an incoming electromagnetic wave. We can see that the expression for the voltage pickup of the loop is substantially different from the expression for the voltage pickup of the vertical wire antenna. In particular, the voltage pickup of the loop improves as the wavelength of the electromagnetic wave in question decreases while the voltage pickup of the vertical wire antenna does not depend on the wavelength of the incoming electromagnetic wave.

In order to be able to make any comparisons to the vertical wire antenna we need to make specifications for a “typical” loop antenna. Let us assume a side length of $s = 1\text{m}$, resulting in a loop that easily fullfills our requirement that it's size needs to be much smaller than $\lambda/4$ for wavelengths bigger than 200m ($f < 1500\text{kHz}$). Setting the number of turns to be $n = 5$, the total wire length is $L = 20\text{m}$. This choice roughly satisfies $L \lll \lambda/4$ for wavelengths bigger than 200m. The first comparison we can now make is the voltage pickup of the vertical wire versus the loop antenna. Using equations (1) and (12) we find that

$$\boxed{\frac{\hat{U}_{\text{VerticalWire}}}{\hat{U}_{\text{Loop}}} = \frac{h\lambda}{4\pi s^2 n}} \tag{13}$$

Substituting the parameters for the vertical wire antenna used so far and the specifications of the loop given above we get

$$\frac{\hat{U}_{\text{VerticalWire}}}{\hat{U}_{\text{Loop}}} = \frac{\lambda}{12.6\text{m}}$$

It becomes obvious that the voltage pickup of the vertical wire antenna greatly exceeds the voltage pickup of the loop antenna for all wavelengths for which the above equations are valid.

The equivalent circuit of the loop antenna is similar to that of the vertical wire antenna, however, there is one major difference: Since the loop does not need a ground connection, there is no ground loss resistance R_{g}. The only loss resistance remaining is the resistance of the wire R_{w}. The resulting equivalent circuit of the loop antenna is shown in figure 5.

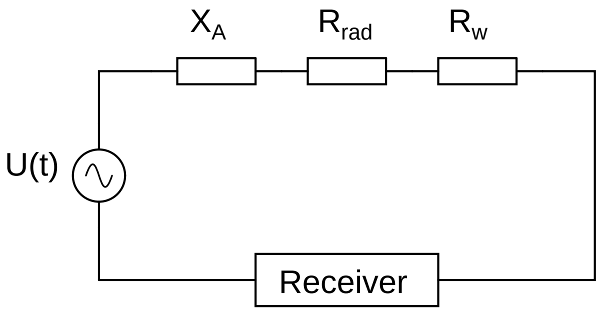

Figure 5: Loop Antenna Equivalent Circuit

Power Transfer From The Lossless Loop Antenna

We are now ready to calculate the maximum power transfer of an ideal (lossless) loop antenna into a receiver. As with the vertical wire antenna, maximum power transfer is obtained under full reactance compensation and impedance matching. That is, the receiver need to provide a reactance $-X_{\mathrm{A}}$ and an (ohmic) input impedance of $R_{\mathrm{in}} = R_{\mathrm{rad}}$. The equivalent circuit of this setup is therefore identical to figure 3 and the maximum power transfer into the receiver is given by (3). Substituting the voltage pickup of the loop antenna (12) into (3) we get

$$P_{\mathrm{in}} = \frac{\pi^2 s^4 n^2 \hat{E}^2}{2\lambda^2 R_{\mathrm{rad}}} \tag{14}$$

The radiation resistance R_{rad} of the loop antenna immediately follows from the definition of the radiation resistance (2) and the total radiated power P_{trp} of the loop antenna when used as a transmitting antenna. The total radiated power of the n-turn loop is [16]

$$P_{\mathrm{trp}} = \frac{8}{3}\pi^3 Z_0 \frac{s^4}{\lambda^4} n^2 \cdot I_{\mathrm{rms}}^2$$

where we have used that [4] $Z_0 = \mu_0 c$ and $\hat{I} = \sqrt{2} \cdot I_{\mathrm{rms}}$ for sinusoidal currents. The radiation resistance of the loop is then by virtue of (2) obtained to be

$$\boxed{R_{\mathrm{rad}} = \frac{8}{3}\pi^3 Z_0 \frac{s^4}{\lambda^4} n^2} \tag{15}$$

Using this result in (14) we finally get

$$\boxed{P_{\mathrm{in}} = \frac{\lambda^2 \hat{E}^2}{\frac{16}{3}\pi Z_0}} \tag{16}$$

which is twice the power transfer of a lossless vertical wire antenna. The reason behind this is that the vertical wire is a (electric) monopole antenna, while the loop is a (magnetic) dipole antenna. The magnitude of the power transfer, however, is still the same as with the vertical wire antenna. Therefore, all general considerations made on the magnitude of the power transfer from the lossless vertical wire antenna also apply here.

Radiation Resistance And Loss Resistance

We will now look at a lossy loop antenna where the wire resistance R_w will play an important role. First however, for comparison with the wire resistance, we need to find out the magnitude of the radiation resistance. By using the loop parameters $n = 5$ and $s = 1\mathrm{m}$ in equation (15) we obtain $R_\mathrm{rad} = 6\mu\Omega$ for $\lambda = 600\mathrm{m}$ ($f = 500\mathrm{kHz}$).

Wire Resistance

We now need to look at the (ohmic) resistance of the wire. Since the multi-turn loop is a system of wires adjacent to each other, it's ohmic resistance is governed by the skin effect [10] as well as the proximity effect [17]. The proximity effect will add to the skin effect and further reduce the effective cross section of the wire for RF currents. Fortunately, it turns out that if the wire spacing is such that the distance between two adjacent turns is at least 5 times the wire diameter, the proximity effect can be neglected against the skin effect [18]. This precondition can easily be fulfilled for the loop antennas we are looking at.

The physical resistance $R_\mathrm{w,phys}$ of a copper wire of length L and wire radius r_w with skin effect taken into account has been given earlier in equation (8). This time however, since the current distribution is uniform along the wire, the wire resistance R_w from the equivalent circuit is equal to the physical resistance $R_\mathrm{w,phys}$ of the wire. We therefore have

$$R_\mathrm{w} = R_\mathrm{w,phys} = 0.71 \cdot 10^{-3}\,\Omega\mathrm{m}^{\frac{1}{2}} \cdot \frac{L}{r_\mathrm{w}}\lambda^{-\frac{1}{2}} \tag{17}$$

Using $L = 5 \cdot 4 \cdot 1\mathrm{m} = 20\mathrm{m}$ and assuming a wire radius of $r_\mathrm{w} = 0.5\mathrm{mm}$ we get $R_\mathrm{w} \approx 1.2\Omega$ for $\lambda = 600\mathrm{m}$. It becomes obvious that the radiation resistance of the loop antenna can be neglected against the wire resistance.

Power Transfer From The Lossy Loop Antenna

With the radiation resistance R_rad being neglectable against the wire resistance R_w, the power transfer into a receiver under full reactance compensation and optimal impedance matching conditions ($R_\mathrm{in} = R_\mathrm{w}$) is

$$\boxed{P_{\text{in}} = \frac{\pi^2 s^4 n^2 \hat{E}^2}{2\lambda^2 R_{\text{w}}}} \quad (18)$$

The above expression is immediately obtained by substituting R_{w} for R_{rad} in equation (14). Using R_{w} from the previous section (17), we get

$$P_{\text{in}} \approx 1738\Omega^{-1}\text{m}^{-\frac{1}{2}} \cdot \frac{r_{\text{w}} s^3 n}{\lambda^{\frac{3}{2}}} \hat{E}^2$$

for a wire made of copper. By using the parameters used so far for the loop antenna ($s = 1\text{m}$, $n = 5$, $r_{\text{w}} = 0.5\text{mm}$) and setting $\lambda = 600\text{m}$ we obtain $P_{\text{in}} = 0.67\text{nW}$ for a relatively weak signal of $\hat{E} = 1500\mu\text{V/m}$ and $P_{\text{in}} = 67\text{nW}$ for a relatively strong signal of $\hat{E} = 15000\mu\text{V/m}$. These figures are considerably below the figures for the power transfer from a lossy vertical wire antenna and, of course, are also nowhere close to the theoretical limit for a lossless loop antenna.

Comparison Of Antenna Performance

The conclusions to draw from the previous sections when it comes to voltage pickup and power transfer into a receiver are: First, with both antenna types the practical power transfer figures are magnitudes below to the theoretical limit. Second, the vertical wire antenna of typical "outdoor" dimensions has the advantage over a loop antenna of typical "indoor" dimensions. Although the loss resistance of a loop antenna is much lower than the loss resistance of a vertical wire antenna (because the loop does not need a ground connection) the vertical wire antenna can provide significantly more power to a receiver. While power transfer into the receiver is crucial for crystal radio sets, regular receivers relying on external power for amplification can benefit from the advantages of the loop antenna which are it's directionality and it being mostly impervious to local electric interference and noise.

Appendix A: Variables And Constants

List of selected variables and constants (SI-units are used throughout this paper)

λ: Wavelength, $\lambda = \frac{c}{f} = \frac{2\pi c}{\omega}$

c: Speed of light in a vacuum and approximately also in air, $c \approx 3 \cdot 10^8 \frac{\text{m}}{\text{s}}$

ω: Angular frequency ($\omega = 2\pi f$)

n: Number of turns of a multi-turn loop

s: Side length of a square loop

h: Physical height of a vertical wire antenna

r_{w}: Wire radius

L: Wire length

μ_0: Magnetic constant, $\mu_0 = 12.56637... \cdot 10^{-7} \frac{\mathrm{Vs}}{\mathrm{Am}}$

μ: Absolute magnetic permeability

ρ: Resistivity

Z_0: Impedance of free space

References

[1] K.R. Sturley, *Radio Receiver Design, JOHN WILEY & SONS*, 1943

[2] http://en.wikipedia.org/wiki/Antenna_(radio)

[3] http://en.wikipedia.org/wiki/Dipole_antenna#Short_dipole

[4] http://en.wikipedia.org/wiki/Impedance_of_free_space

[5] http://en.wikipedia.org/wiki/Monopole_antenna

[6] U.A. Bakshi et. al. , *Antennas And Wave Propagation, Technical Publications Pune*, 2008

[7] International Telecommunication Union (ITU), *Final Acts of the Regional Administrative LF/MF Broadcasting Conference Geneva 1975*

[8] http://en.wikipedia.org/wiki/Sound_intensity

[9] http://www.sengpielaudio.com/TabelleDerSchallpegel.htm

[10] http://en.wikipedia.org/wiki/Skin_effect

[11] http://en.wikipedia.org/wiki/Resistivity

[12] http://en.wikipedia.org/wiki/Magnetic_permeability

[13] *Military Handbook 419A, Grounding Bonding and Shielding for Electronic Equipments and Facilities*, 1987

[14] http://en.wikipedia.org/wiki/Faraday's_law_of_induction

[15] http://en.wikipedia.org/wiki/Electromagnetic_wave

[16] Jochen Bauer, http://www.radiomuseum.org/forum/signal_levels_of_loop_antennas_for_am_home_transmitters.html

[17] http://en.wikipedia.org/wiki/Proximity_effect_(electromagnetism)

[18] Glenn Smith, *The Proximity Effect In Systems Of Parallel Conductors and Electrically Small Multiturn Loop Antennas, Office of Naval Research Technical Report No. 624*, 1971

[19] Karalis, Joannopoulos, Soljačić *Efficient wireless non-radiative mid-range energy transfer, Annals of Physics 323*, 2008