Chapter 21

CIRCUIT ELEMENTS OF LARGER CROSS SECTIONS WITH PARALLEL AXES

Solenoid and Circular Filament.

Let n_1 = the winding density of the solenoid,
A = radius of solenoid,
b = axial length of solenoid,
a = radius of the circular filament,
ρ = distance between axes,
u = distance between centers measured along the axes (see Fig. 56).

Placing

$$d_1 = u + \frac{b}{2}, \qquad d_2 = u - \frac{b}{2},$$

$$r_1 = \sqrt{\rho^2 + d_1^2}, \quad r_2 = \sqrt{\rho^2 + d_2^2},$$

$$\mu_1 = \frac{d_1}{r_1}, \qquad \mu_2 = \frac{d_2}{r_2},$$

it is found that

$$M = 0.001\pi^2 a^2 A^2 n_1 \left[\frac{V_2}{r_2^2} - \frac{V_1}{r_1^2}\right], \quad (182)$$

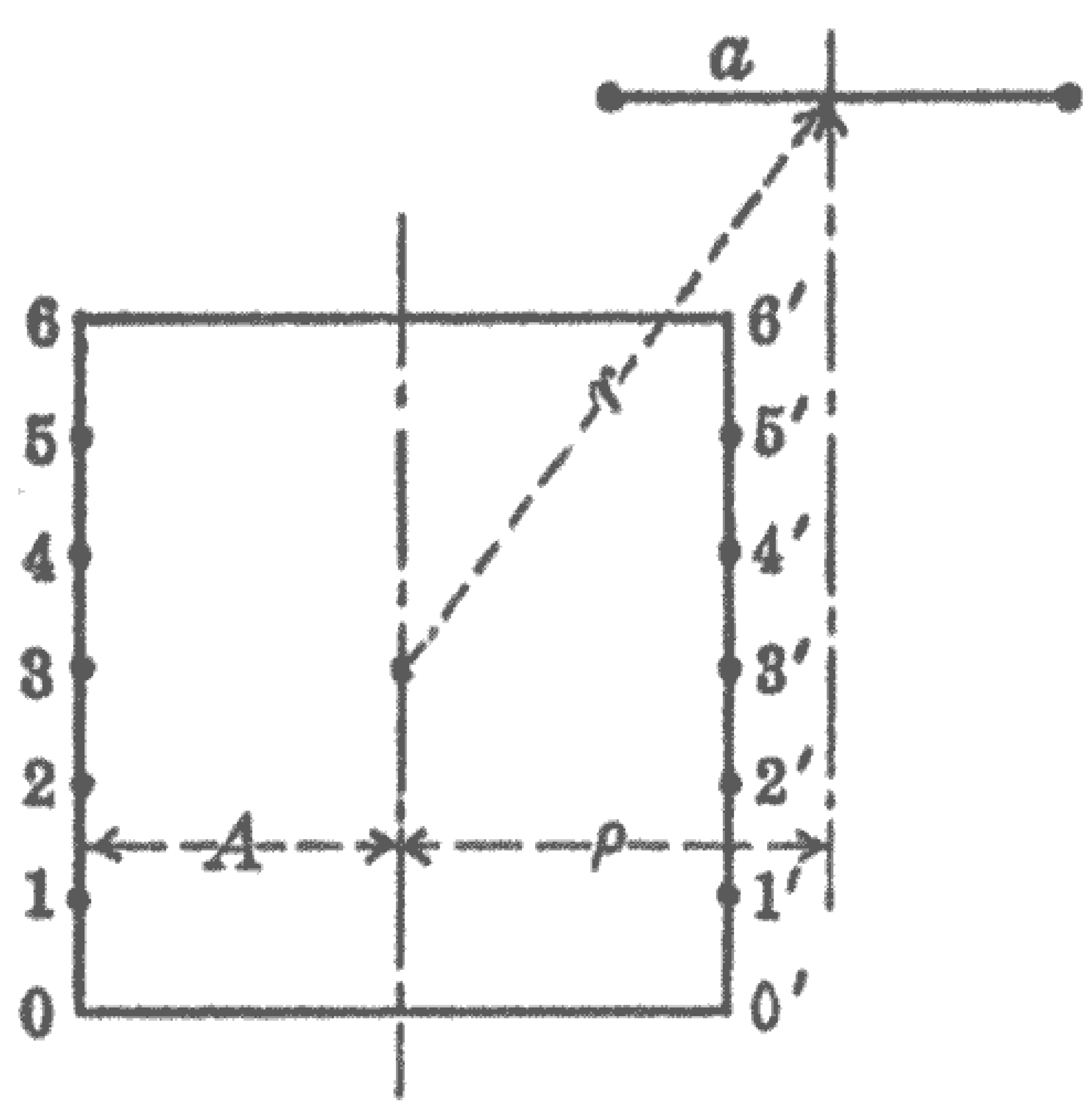

FIG. 56

in which V_2 and V_1 are found by substituting, respectively, μ_2, r_2 and μ_1, r_1, for μ, r in the expression

$$V = \mu\left[1 - \frac{3}{4}K_1\frac{A^2}{r^2}\frac{P_3(\mu)}{\mu} + \frac{5}{8}K_2\frac{A^4}{r^4}\frac{P_5(\mu)}{\mu} - \frac{35}{64}K_3\frac{A^6}{r^6}\frac{P_7(\mu)}{\mu} + \cdots\right]. \quad (183)$$

The general term of this series is

$$(-1)^n\frac{3\cdot 5\cdot 7\cdots(2n+1)}{4\cdot 6\cdot 8\cdots(2n+2)}K_n\left(\frac{A}{r}\right)^{2n}\frac{P_{2n+1}(\mu)}{\mu}.$$

The factors K_1, K_2, and K_3 may be interpolated from Table 47 as a function of the ratio $\alpha^2 = \dfrac{a^2}{A^2}$. The zonal harmonic functions $\dfrac{P_{2n+1}(\mu)}{\mu}$ may be obtained from Auxiliary Table 3 (page 238).

TABLE 47. VALUES OF K_n IN FORMULAS (183) AND (185)

Interpolation may be avoided by using the following formulas directly:

$$K_1 = 1 + \alpha^2, \qquad K_3 = 1 + 6\alpha^2 + 6\alpha^4 + \alpha^6.$$

$$K_2 = 1 + 3\alpha^2 + \alpha^4, \qquad K_4 = 1 + 10\alpha^2 + 20\alpha^4 + 10\alpha^6 + \alpha^8.$$

In general $K_n = F(-n-1, -n, 2, \alpha^2)$, where F is the hypergeometric series.

α^2	K_1	K_2	Δ_1	K_3	Δ_1	Δ_2	K_4	Δ_1	Δ_2	Δ_3
0	1.0	1.00		1.000			1.000			
			31		661			1210		
0.1	1.1	1.31		1.661		126	2.210		462	
			33		787			1672		62
.2	1.2	1.64		2.448		132	3.882		524	
			35		919			2196		67
.3	1.3	1.99		3.367		138	6.078		591	
			37		1057			2787		69
.4	1.4	2.36		4.424		144	8.866		660	
			39		1201			3447		70
0.5	1.5	2.75		5.625		150	12.312		730	
			41		1351			4177		73
.6	1.6	3.16		6.976		156	16.490		803	
			43		1507			4980		76
.7	1.7	3.59		8.483		162	21.470		879	
			45		1669			5859		78
.8	1.8	4.04		10.152		168	27.330		957	
			47		1837			6816		81
0.9	1.9	4.51		11.989		174	34.146		1038	
			49		2011			7854		
1.0	2.0	5.00		14.000			42.000			
$\Delta_1 = 0.1$		$\Delta_2 = 2$		$\Delta_3 = 6$			$\Delta_4 = 2.7$			

By applying the principle of interchange of lengths, it is evident that the mutual inductance of a solenoid of radius a, length b, and a circular filament of radius A is the same as the mutual inductance of the solenoid of radius A, length b, and the circular filament of radius a in Fig. 56, provided ρ and u are the same in both cases. Therefore, the general formula (182) may be used, whichever element has the larger radius A.

Example 76:

$$a = A = 10 \text{ cm.}, \qquad u = 20 \text{ cm.},$$

$$b = 12, \qquad \rho = 20.$$

From Table 47, for $\alpha^2 = \frac{a^2}{A^2} = 1$, there are found $K_1 = 2$, $K_2 = 5$, $K_3 = 14$. From the given data,

$$d_2 = 14, \qquad d_1 = 26,$$

$$r_2^2 = 596, \qquad r_1^2 = 1076,$$

$$\mu_2 = 0.57348, \qquad \mu_1 = 0.79263,$$

and from Auxiliary Table 3

$$\frac{P_3(\mu_2)}{\mu_2} = -0.6780, \qquad \frac{P_3(\mu_1)}{\mu_1} = 0.0706,$$

$$\frac{P_5(\mu_2)}{\mu_2} = -0.1523, \qquad \frac{P_5(\mu_1)}{\mu_1} = -0.5140,$$

$$\frac{P_7(\mu_2)}{\mu_2} = 0.5561, \qquad \frac{P_7(\mu_1)}{\mu_1} = -0.2656.$$

Using these values we find in (183)

$$V_2 = 0.57348(1 + 0.17064 - 0.01340 - 0.02011)$$

$$V_2/r_2^2 = 0.6521 \div 596 = 0.0010942,$$

$$V_1 = 0.79263(1 - 0.00984 - 0.01387 + 0.00163)$$

$$V_1/r_1^2 = 0.7751 \div 1076 = 0.0007204,$$

so that

$$M = 0.001\pi^2(100)^2 n_1(0.0003738)$$

$$= 0.03689 n_1 \ \mu\text{h}.$$

The general formula (183) converges well only if $\frac{A^2}{r_n^{\ 2}}$ is small, and furthermore, the mutual inductance is given by the difference of two terms each of which has to be calculated with a greater degree of precision than is required in the result. Unless the distances r_1 and r_2 are considerably larger than the sum of the radii, the accuracy may not be sufficient.

In such cases the Rayleigh quadrature formula (page 11), or some other method of averaging may have to be applied. For equal radii this is not difficult, as may be illustrated for the case treated in example 76.

Example 77: The degree of convergence in the preceding example leaves something to be desired, especially with respect to the calculation of V_2. To test the accuracy of the result, suppose circular filaments to be selected at equal intervals along the length of the solenoid (Fig. 56), in the positions 00′, 11′ ⋯ 66′. The mutual inductance of each of these filaments and the given circular filament will be calculated.

Since the radii are all the same, Table 43 for equal circles with parallel axes may be employed. The main results of the calculation follow:

Circle	00′	11′	22′	33′	44′	55′	66′
d_n	26.0	24.0	22.0	20.0	18.0	16.0	14.0
r_n^2	1076.0	976.0	884.0	800.0	724.0	656.0	596.0
$\frac{2A}{r_n}$	0.60999	0.64018	0.67266	0.70710	0.74330	0.78088	0.81922
$\mu_n = \frac{d_n}{r_n}$	0.79300	0.76823	0.73995	0.70710	0.66896	0.62470	0.57346

The values of F are interpolated from Table 43 for these values of $\frac{2A}{r_n}$ and μ_n and the values of f from Table 17 for coaxial circles using $\frac{\text{diameter}}{\text{distance}} = \frac{2A}{r_n}$.

	00′	11′	22′	33′	44′	55′	66′
F	0.6197	0.5852	0.5471	0.5040	0.4517	0.3885	0.3127
$1000f$	0.4386	0.4961	0.5621	0.6362	0.7187	0.8091	0.9060
$1000Ff$	0.2718	0.3075	0.3075	0.3206	0.3248	0.3143	0.2833

The mutual inductance m of each circle with the given circular filament is $AFfn_1\,dx$ and the total desired mutual inductance is found by integrating this over the length of the solenoid. The integration may be obtained by Simpson's rule, using these calculated values of the integrand. The interval of integration is $\frac{1}{6}$ of 12, or 2 cm.

Even	*Odd*	*Extremes*		
0.2718	0.2903	0.2718	$2 \times 1.1874 =$	3.3748
0.3075	0.3206	0.2833	$4 \times 0.9252 =$	3.7008
0.3248	0.3143			
0.2833		0.5551 = Sum		6.0756
	0.9252 = Sum			−0.5551
1.1874 = Sum				
				5.5205
			times $\frac{1}{3}$ of 2 cm. =	3.6804

$$M = 0.001(10)(3.6804)n_1 = 0.036804n_1\ \mu\text{h}.$$

This value is more accurate than that calculated by formula (182). This example is a favorable case for this method, since the calculated points differ only slowly. The increase of m, due to decreasing distance between circles, is offset by the decrease in m, due to decreasing μ.

For this case also the Rayleigh formula is favorable. For this problem this becomes

$$M = \tfrac{1}{6}(4m_{33} + m_{00} + m_{66})(\text{number of turns})$$
$$= 0.3061(0.001)(10)(12n_1)$$
$$= 0.036732n_1\ \mu\text{h}.$$

This is nearly as accurate as the preceding value and requires the calculation of only three circles, whereas seven were necessary for the other.

If the radii of the solenoid and circular filament are not equal, the matter is complicated. For values not very different, instead of the parameter $\frac{2A}{r_n}$, the ratio of the mean diameter of coil and circle to r_n may be used with a moderate accuracy. Otherwise, the required mutual inductances of the unequal circles with parallel axes that enter in the calculation should be obtained by the graphical method described on page 187.

Solenoids with Parallel Axes. Expressions for the mutual inductance of single-layer coils with parallel axes have been given by Dwight [92] and Purssell and by Clem.[93] It is easy to show that the following formula may be derived from these and it is in an especially convenient form for numerical calculations.

The two coils of radii a and A are shown in Fig. 57. Their lengths x and l are taken as equal to the number of turns times the pitch of the windings. Accordingly, the winding densities n_1 and n_2 are, respectively, $n_1 = \frac{N_1}{x}$ and $n_2 = \frac{N_2}{l}$.

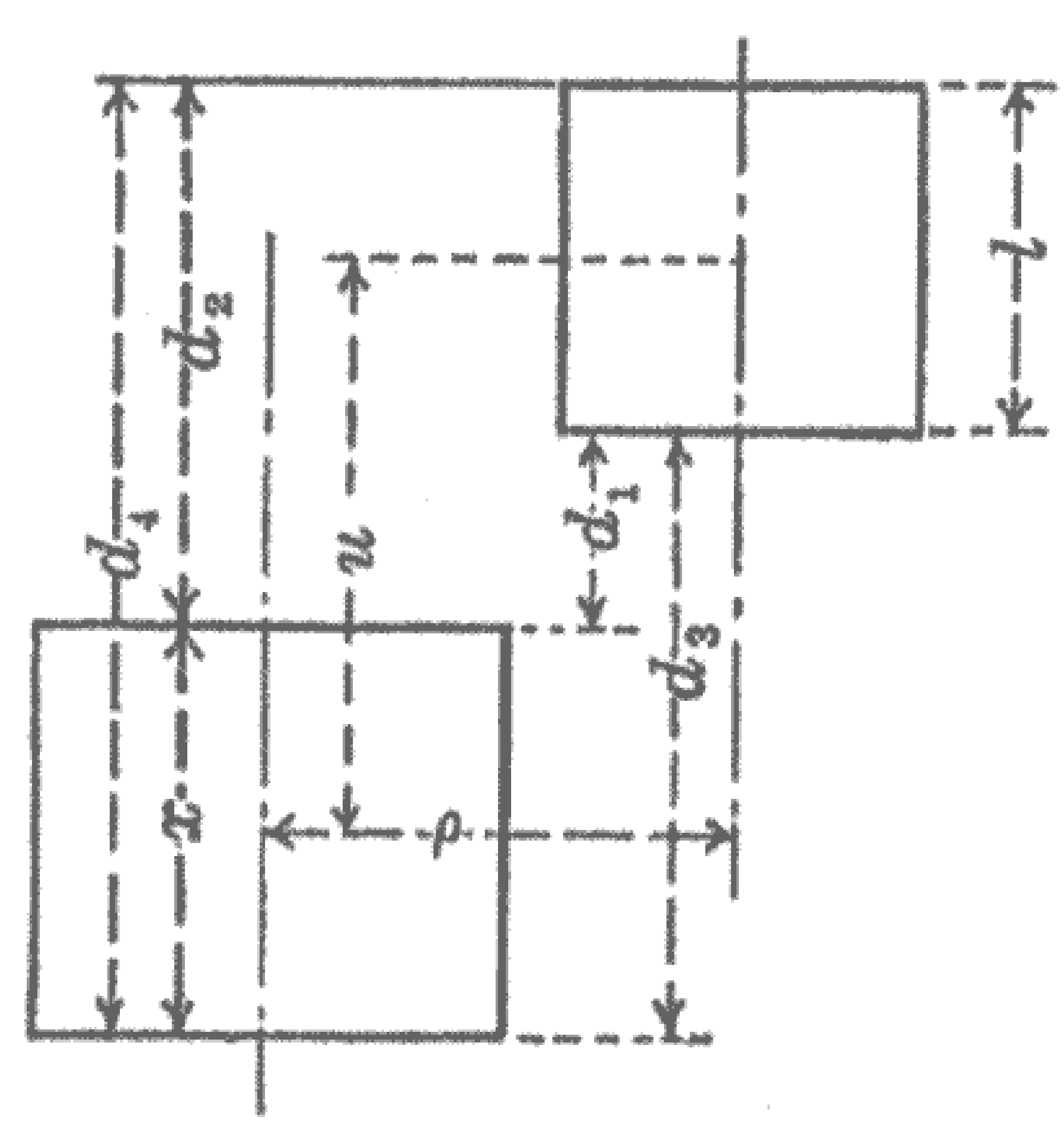

Fig. 57

Let ρ = distance between the axes, and calculate the four distances d_n between the ends of the coils shown in Fig. 57.

$$d_1 = u - \left(\frac{x+l}{2}\right), \quad d_3 = u + \left(\frac{x-l}{2}\right),$$

$$d_2 = u + \left(\frac{l-x}{2}\right), \quad d_4 = u + \left(\frac{x+l}{2}\right),$$

in which u = axial distance between the centers of the coils (see Fig. 57).

From these distances are to be calculated the four radii vectors $r_n = \sqrt{\rho^2 + d_n{}^2}$ and the four cosines $\mu_n = \frac{d_n}{r_n}$.

Then,

$$M = 0.001\pi^2 a^2 A^2 n_1 n_2 \left[\frac{X_1}{r_1} - \frac{X_2}{r_2} - \frac{X_3}{r_3} + \frac{X_4}{r_4}\right] \mu\text{h}, \tag{184}$$

in which

$$X_n = \left[1 - \frac{1}{4} K_1 \frac{A^2}{r_n^{\,2}} P_2(\mu_n) + \frac{1}{8} K_2 \frac{A^4}{r_n^{\,4}} P_4(\mu_n) \right.$$

$$\left. - \frac{5}{64} K_3 \frac{A^6}{r_n^{\,6}} P_6(\mu_n) + \frac{7}{128} K_4 \frac{A^8}{r_n^{\,8}} P_8(\mu_n) - \cdots\right]. \tag{185}$$

The constants K_1, K_2, K_3, and K_4 are functions of $\alpha^2 = \dfrac{a^2}{A^2}$ and may be calculated from the formulas

$$K_1 = 1 + \alpha^2, \qquad K_2 = 1 + 3\alpha^2 + \alpha^4,$$

$$K_3 = 1 + 6\alpha^2 + 6\alpha^4 + \alpha^6, \quad K_4 = 1 + 10\alpha^2 + 20\alpha^4 + 10\alpha^6 + \alpha^8,$$

or interpolated from Table 47. The zonal harmonics $P_{2n}(\mu_n)$ may be interpolated from Auxiliary Table 3 (page 238).

If the coils are overlapping, some of the distances d_n may be regarded as negative. This does not, however, affect the values of r_n and the zonal harmonics $P_{2n}(-\mu) = P_{2n}(\mu)$, so that the signs of the d_n are immaterial in the formula for the mutual inductance.

For the special case that the coils have their bases in the same plane

$$d_1 = -x \qquad d_3 = 0,$$

$$d_2 = \pm (x - l), \quad d_4 = l;$$

and for equal coils resting on the same plane

$$x = l, \qquad d_2 = 0,$$

$$d_1 = d_4 = x, \quad d_3 = 0;$$

and formula (184) becomes

$$M = 0.002\pi^2 a^2 A^2 n_1 n_2 \left[\frac{X_1}{r_1} - \frac{X_2}{r_2}\right] \mu\text{h}. \tag{186}$$

The convergence of (185) is better the smaller the values of $\left(\dfrac{A}{r_n}\right)^2$, and each r_n must be greater than $(A + a)$. The principle of interchange of the lengths holds, but since the d_n, r_n, and μ_n are thereby unchanged, no improvement of the convergence is thereby obtained.

Formula (184) is of such a form that the individual terms have to be calculated to a higher degree of accuracy than is attainable in the result. This disadvantage is especially acute for distant coils, but for such cases the convergence of formula (185) is good.

The writer finds that for the special case of loosely coupled coils of equal radii A and equal length B the following series formula may be used:

$$M = 0.002\pi^2 \frac{A^4 n_1 n_2 \beta^2}{R} [P_2(\mu) + \beta^2 P_4(\mu) + \beta^4 P_6(\mu) + \beta^6 P_8(\mu) + \cdots$$

$$- \frac{1}{2}\frac{A^2}{R^2}\{6P_4(\mu) + 15\beta^2 P_6(\mu) + 28\beta^4 P_8(\mu) + \cdots\}$$

$$+ \frac{25}{8}\frac{A^4}{R^4}\{3P_6(\mu) + 14\beta^2 P_8(\mu) + \cdots\}$$

$$- \frac{245}{8}\frac{A^6}{R^6}\{P_8(\mu) + \cdots\} + \cdots] \ \mu\text{h}, \tag{187}$$

in which the distance between the axes is assumed to be ρ and the distance between centers $R = \sqrt{\rho^2 + u^2}$. The argument of the zonal harmonics is $\mu = \frac{u}{R}$, and $\beta = \frac{B}{R}$. The convergence is better the smaller the space ratios $\frac{A}{R}$ and β.

Example 78: The mutual inductance of two equal single-layer coils of radii $A = 5$, length $x = 10$, and winding density 20 turns per cm. will be calculated by formula (184). The distance between the axes will be taken as $\rho = 10$ and the axial distance between their nearer ends as 5 cm. That is, $u = 15$.

Then

$$d_1 = 5, \qquad d_2 = d_3 = 15, \qquad d_4 = 25,$$

$$r_1^2 = 125, \qquad r_2^2 = r_3^2 = 325, \qquad r_4^2 = 725.$$

Since the radii are equal, Table 47 gives for $\alpha^2 = 1$,

$$K_1 = 2, \quad K_2 = 5, \quad K_3 = 14, \quad K_4 = 42.$$

We find also

$$\mu_1 = 0.44721, \quad \mu_2 = \mu_3 = 0.83205, \quad \mu_4 = 0.92848,$$

and for these values Auxiliary Table 3 gives

$P_2(\mu_1) = -0.1999,$	$P_2(\mu_2) = 0.5385,$	$P_2(\mu_4) = 0.7931,$
$P_4(\mu_1) = -0.2000,$	$P_4(\mu_2) = -0.1243,$	$P_4(\mu_4) = 0.3936,$
$P_6(\mu_1) = 0.3280,$	$P_6(\mu_2) = -0.4147,$	$P_6(\mu_4) = -0.0366.$
$P_8(\mu_1) = -0.2000,$		

The series for the X_n are

X_1	$X_2 = X_3$	X_4
1.01999	1	1
−0.00500	−0.02071	−0.01367
−0.00287	−0.00046	+0.00029
−0.00072	+0.00021	−0.00002
1.01139	0.97904	0.98662

$$\frac{X_1}{r_1} = 0.090461 \quad \frac{X_2}{r_2} = 0.054307$$

$$\frac{X_4}{r_4} = \underline{0.036642}$$

$$0.127103 - 0.108614 = 0.018489$$

$$M = 0.001\pi^2(25)^2(20)^2(0.018489) = 45.62 \ \mu\text{h}.$$

In order to obtain a three figure accuracy in the result, it is necessary that the separate terms shall be accurate to the fourth significant figure. The value of X_1 should be calculated to include one more term to assure this accuracy.

Example 79: To illustrate the use of formula (187) the solution will be found for the case of two equal loosely coupled coils for which the given constants are

$$A = 5, \qquad \rho = 15, \qquad n_1 = 20,$$

$$B = 5, \qquad u = 10, \qquad n_2 = 20.$$

The distance between centers is $R = \sqrt{15^2 + 10^2} = \sqrt{325}$. The space ratios are $\beta = \frac{B}{R} = \frac{5}{\sqrt{325}}$ and $\frac{A}{R} = \frac{5}{\sqrt{325}}$. The zonal harmonics for $\mu = \frac{u}{R} = \frac{10}{\sqrt{325}} =$ 0.55470, taken from Auxiliary Table 3, are

$$P_2(\mu) = -0.0385, \qquad P_6(\mu) = 0.2634,$$

$$P_4(\mu) = -0.3640, \qquad P_8(\mu) = 0.0871.$$

It will be necessary to obtain a more precise value of $P_2(\mu)$ by the defining relation $P_2(\mu) = \frac{1}{2}(3\mu^2 - 1)$, which leads to the value -0.0384615.

The four terms in the brackets of formula (187) yield for this case $-0.06486 + 0.07176 + 0.01635 - 0.00121 = 0.02205$.

Substituting in formula (187), $M = 0.4643 \ \mu$h.

For this case, formula (184) requires each of the quantities X_n to be calculated accurately to seven significant figures to give a four figure accuracy in the result. The series given for the X_n are not sufficiently convergent to allow this without further terms. The calculated M comes out 0.472 μh.

A further method of attack, applicable to loosely coupled solenoids of unequal radii, where formula (187) cannot be used, is to integrate formula (182) for solenoid and eccentric circle over one of the solenoids. Making the calculation for a number of equally spaced turns, the integration may be accomplished mechanically.

To illustrate the process the solution may be found for the problem just considered. Seven circles, a, b, c, $\cdots$ g equally spaced axially along one of the coils are taken. The mutual inductance m of each circle and the other solenoid is calculated by formula (182) for solenoid and circle with parallel axes. The calculated mutual inductances in abhenrys are -38.26, -18.23, -3.29, 9.10, 18.07, 24.85, and 30.79. The distance between consecutive circles is $\frac{5}{6}$ cm. Summing by Simpson's rule and multiplying by the winding density $n_2 = 20$, the value $M = 0.4721$ μh is found, which agrees with the value by (187) to about 1 per cent.

Example 80: The last described method is more accurate for still more loosely coupled coils. Assume two equal coils with

$$A = 2.5, \qquad u = 0, \qquad n_1 = 25,$$
$$B = 5, \qquad \rho = 25, \qquad n_2 = 25.$$

Here $R = 25$, $\dfrac{A^2}{R^2} = 0.01$, $\beta^2 = 0.04$, and $\mu = 0$. The zonal harmonics for this value, taken from Auxiliary Table 3, are

$$P_2(\mu) = -0.5, \quad P_4(\mu) = 0.375, \quad P_6(\mu) = -0.3125, \quad P_8(\mu) = 0.2734.$$

By (187)

$$M = 0.002\pi^2 \frac{(25)^2(2.5)^4(0.04)}{25} [-0.48448 - 0.01037 - 0.00002]$$
$$= -0.38159 \ \mu\text{h}.$$

The convergence of (187) is excellent.

To apply the Rayleigh quadrature method, calculate by formula (182) the mutual inductance of one solenoid on the three circles taken at the ends of the other solenoid and at its midsection. From the symmetry of this arrangement, the value will be the same for each end section:

$$u = 2.5, \qquad d_2 = 0, \qquad d_1 = 5.$$
$$\mu_2 = 0, \qquad \mu_1 = 0.19611,$$

The zonal harmonics $P_{2n+1}(\mu)$ are all zero for $\mu = 0$, so that the value of $\dfrac{V_2}{r_2^2}$ in (182) is zero. The calculated value of m_e for the end circles comes out -2.968 abhenrys. For the circle at the midsection

$$d_2 = -2.5, \quad d_1 = 2.5, \quad \mu_2 = -\mu_1.$$

Since $P_{2n+1}(-\mu_1) = -P_{2n+1}(\mu_1)$, the two terms of (182) give $-2\dfrac{V_1}{r_1^2}$, and the mutual inductance m_c is -3.107 abhenrys.

The Rayleigh formula for this case gives

$$M = \frac{N_2}{6}(4m_c + 2m_e)10^{-3} \ \mu\text{h}$$
$$= \frac{125}{1000}\left[\frac{4(-3.107) + 2(-2.968)}{6}\right] = -0.3826 \ \mu\text{h}.$$

The value obtained by multiplying the value of the mutual for the midcircle by N_2 is -0.386, which is an approximation that may often be sufficient.

Solenoids with Parallel Axes Having Zero Mutual Inductance. The preceding examples have included cases where the mutual inductance may have either sign, which suggests the possibility of placing the coils so as to have zero mutual inductance. Such an arrangement is used in the familiar case of the coils in a neutrodyne circuit.

To determine how the coils should be placed we may employ the series formula (187) for equal solenoids. Imposing the condition that $M = 0$, the corresponding value of $\mu = \dfrac{u}{R}$ may be found by successive approximations. This is readily accomplished for given numerical data making use of Auxiliary Table 3 for values of the zonal harmonics.

Example 81: Assume coil dimensions and spacing such that $\beta = \dfrac{1}{4}, \dfrac{A}{R} = \dfrac{1}{4}$, that is, the coil length is one half of the diameter, and the distance between centers is four times the length of the coil.

Substituting these values in the series of formula (187), the necessary condition for zero mutual inductance is

$$P_2(\mu) - \tfrac{1}{8}P_4(\mu) + \tfrac{23}{2048}P_6(\mu) + \tfrac{1}{32768}P_8(\mu) = 0.$$

Using the first term and Auxiliary Table 3 it is evident that μ must be about 0.55. Calculating the above sum for several values of μ near 0.55 the sum is found to be as follows:

μ	Sum
0.53	−0.0337
.54	−0.0161
.55	+0.0017
0.56	0.0196.

The value of $\mu = 0.549$ is, closely, the solution. Denoting by θ the angle between the line joining the coil centers and the direction of the axes, this value of μ corresponds to $\theta = 56° \, 42'$.

If the coil is very short, axially, $\beta = 0$, and the corresponding value of μ is 0.533 or $\theta = 57° \, 48'$. This is checked by the use of Table 43 for eccentric circles, from which the value $\mu = 0.536$ is found.

For longer coils, with $\beta = \tfrac{1}{3}$, with the same value of $\dfrac{A}{R}$ as before, it works out that $\mu = 0.561, \theta = 55° \, 53'$.

For coils far apart so that $\beta \approx 0$ and $\dfrac{A}{R} \approx 0$, the limiting value is $\mu = 0.577$, or $\theta = 54° \, 44'$.

It is evident, therefore, that except for coils close together, the condition for zero mutual inductance is not very critical.

Solenoid and Coil of Rectangular Cross Section with Parallel Axes. A general formula, derived from one by Dwight and Purssell,[92] is the following, in which the nomenclature is that of Fig. 58, and n_1 is the winding density of the solenoid and N_2 the total number of turns on the coil.

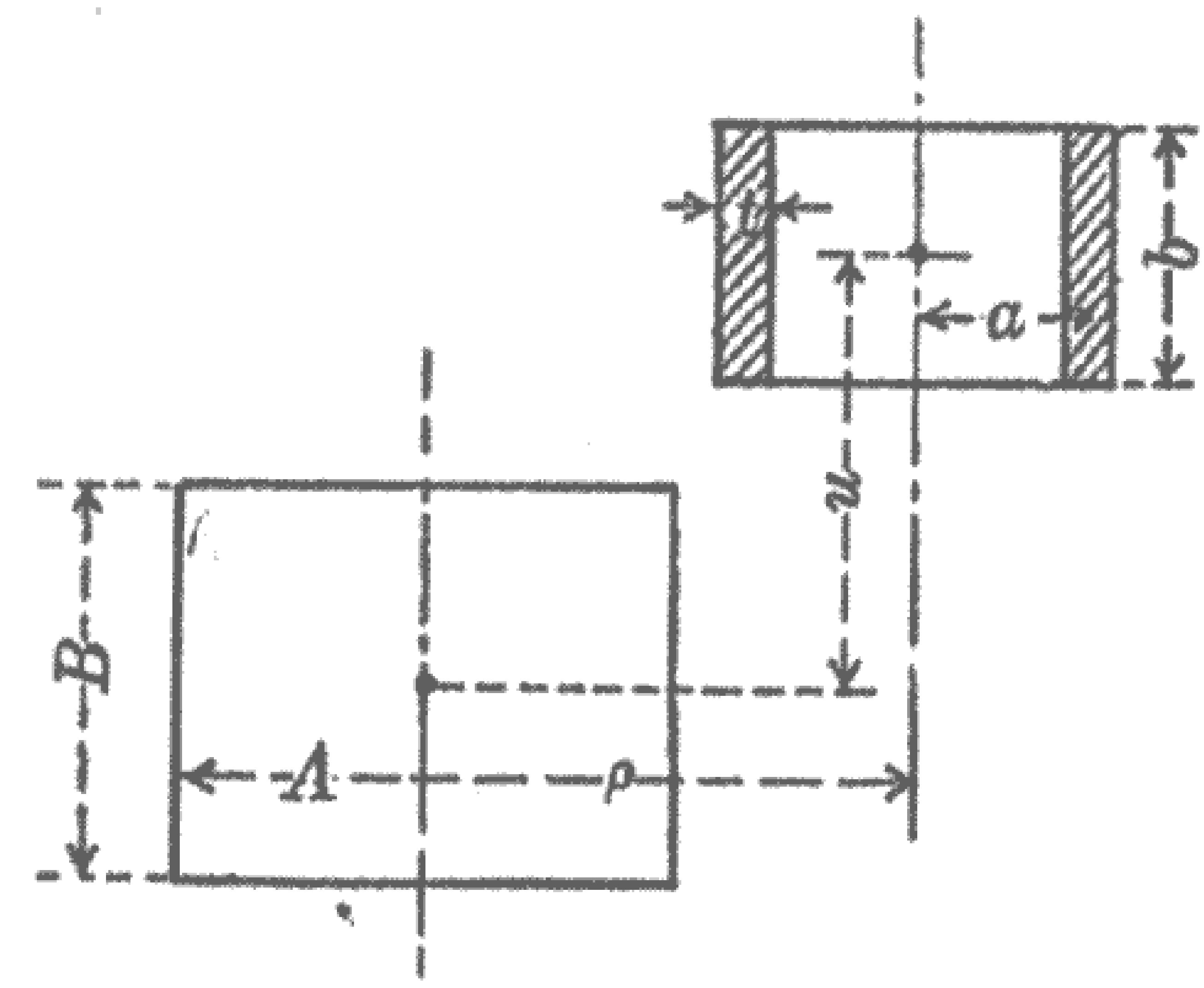

FIG. 58

$$M = 0.001\pi^2 A^2 a^2 n_1 \frac{N_2}{b}\left[\frac{Y_1}{r_1} - \frac{Y_2}{r_2} - \frac{Y_3}{r_3} + \frac{Y_4}{r_4}\right] \mu\text{h}, \tag{188}$$

in which

$$\begin{aligned} Y_m = \Big\{ & t_2 - \frac{1}{4}\frac{A^2}{r_m{}^2}\left(t_2 + \frac{a^2}{A^2}t_4\right)P_2(\mu_m) \\ & + \frac{1}{8}\frac{A^4}{r_m{}^4}\left(t_2 + 3\frac{a^2}{A^2}t_4 + \frac{a^4}{A^4}t_6\right)P_4(\mu_m) \\ & - \frac{5}{64}\frac{A^6}{r_m{}^6}\left(t_2 + 6\frac{a^2}{A^2}t_4 + 6\frac{a^4}{A^4}t_6 + \frac{a^6}{A^6}t_8\right)P_6(\mu_m) \\ & + \frac{7}{128}\frac{A^8}{r_m{}^8}\left(t_2 + 10\frac{a^2}{A^2}t_4 + 20\frac{a^4}{A^4}t_6 + 10\frac{a^6}{A^6}t_8 + \frac{a^8}{A^8}t_{10}\right)P_8(\mu_m) \\ & - \cdots \Big\}, \end{aligned} \tag{189}$$

with

$$r_m{}^2 = d_m{}^2 + \rho^2, \qquad \mu_m = \frac{d_m}{r_m},$$

$$d_1 = u - \left(\frac{b+B}{2}\right), \quad d_3 = u + \left(\frac{B-b}{2}\right),$$

$$d_2 = u + \left(\frac{b-B}{2}\right), \quad d_4 = u + \left(\frac{B+b}{2}\right).$$

The coefficients t_2, t_4, t_6, etc., are functions of the ratio τ of the thickness of the coil and its mean radius and may be obtained from Table 48, where