Inductively Coupled Circuits

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13.1. Fundamentals of Inductively Coupled Circuits. Included in this section are the basic definitions, equations, and equivalent circuits required for the detailed analysis of inductively coupled air-core and iron-core circuits.

13.1a. Coefficient of Coupling. In Fig. 13.1 a current in either the primary or secondary winding will produce magnetic flux. Depending on the orientation of the two windings, a certain portion of the total flux will link both windings. The dimensionless factor k, known as the *coefficient of coupling,* is equal to the ratio of the flux common to both windings to the total generated flux. If the coefficient of coupling k is equal to 0.5 or greater, the coils are usually said to be closely coupled, and if k is equal to 0.01 or smaller, the coils are said to be loosely coupled.

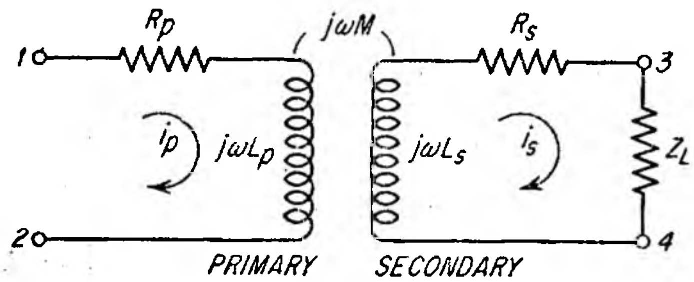


FIG. 13.1. Inductively coupled circuit.

13.1b. Mutual Inductance. Any two windings which have common flux are said to have a *mutual inductance M.* Mutual inductance in henrys can be determined from either Eq. (13.1), (13.2), or (13.3).

$$M = \frac{N_s \phi_{21} 10^{-8}}{i_p} \tag{13.1}$$

where N_s = number of secondary turns
ϕ_{21} = lines of flux common to both windings which are produced by primary current
i_p = primary current, amp

$$M = \frac{N_p \phi_{12} \times 10^{-8}}{i_s} \tag{13.2}$$

where N_p = number of primary turns
ϕ_{12} = lines of flux common to both windings which are produced by secondary current
i_s = secondary current, amp

$$M = k\sqrt{L_p L_s} \tag{13.3}$$

where L_p = primary inductance, henrys
L_s = secondary inductance, henrys

Mutual inductance can be determined experimentally for any two windings by measuring the inductances of the two coils when connected series-aiding (flux fields in the same direction) and series-opposing (flux fields opposing) and dividing the difference in the two measurements by 4.

13.1c. Leakage Inductance. Leakage inductance exists if all the flux produced by a current in one winding does not link the other winding. Primary and secondary leakage inductances in henrys can be determined from Eqs. (13.4) and (13.5).

$$L_{\text{pri. leak.}} = \frac{N_p\phi_1' \times 10^{-8}}{i_p} \tag{13.4}$$

$$L_{\text{sec. leak.}} = \frac{N_s\phi_2' \times 10^{-8}}{i_s} \tag{13.5}$$

where ϕ_1' = lines of flux produced by primary current which do not link secondary winding

ϕ_2' = lines of flux produced by secondary current which do not link primary winding

The sum of the primary and secondary leakage inductances referred to the primary and secondary terminals can be determined from Eqs. (13.6) and (13.7)

$$L_p' = 2(1 - k)L_p \tag{13.6}$$

$$L_s' = 2(1 - k)L_s \tag{13.7}$$

where L_p' = sum of primary and secondary leakage inductances referred to primary terminals

L_s' = sum of primary and secondary leakage inductances referred to secondary terminals

The measured inductance L_1 of the primary with the secondary short-circuited and L_2 of the secondary with the primary short-circuited are given by Eqs. (13.8) and (13.9).

$$L_1 = L_p(1 - k^2) \tag{13.8}$$

$$L_2 = L_s(1 - k^2) \tag{13.9}$$

Equations (13.8) and (13.9) are very nearly equal to Eqs. (13.6) and (13.7) if the value of k approaches unity. Therefore, if k is sufficiently large, the measurements L_1 and L_2 will be very nearly equal to the true values of L_p' and L_s'. The measurements are less than the true values of L_p' and L_s' by approximately 2½ per cent if $k = 0.95$, 5 per cent if $k = 0.90$, and 11 per cent if $k = 0.80$. All short-circuit measurements should be made at frequencies low enough to minimize the effects of distributed capacitance. Where k is either not known or is not large enough to permit the determination of L_p' and L_s' by the short-circuit measurements, it is necessary to measure L_p, L_s, and M, so as to permit the determination of k from Eq. (13.3) and then to calculate L_p' and L_s' from Eqs. (13.6) and (13.7). L_p and L_s should be measured with the secondary and primary respectively open-circuited.

13.1d. Analysis of Air-core and Iron-core Coupled Circuits. Neglecting core losses, the equations for air-core and iron-core inductively coupled circuits are identical; therefore the two types of circuits can be analyzed in the same manner.

If the coupling coefficient is unity and if the copper losses, core losses, and winding capacitance are equal to zero, the voltage ratio will be equal to the turns ratio. These conditions represent the ideal iron-core power transformer. In general, all of these conditions are not satisfied, and the voltage ratio will not be equal to the turns ratio; hence it is usually necessary to make a complete circuit calculation.

For iron-core circuits which have coupling coefficients very nearly equal to unity, simplified equivalent circuits can be employed (see Sec. 14.2). The equivalent circuits discussed in this section, however, are for air-core transformers and iron-core transformers having coupling coefficients which are much less than unity.

Equivalent Circuits. To analyze an inductively coupled circuit, the secondary circuit can be referred to the primary circuit. This results in an equivalent primary circuit which permits the determination of the primary current. The secondary circuit can also be analyzed with the aid of an equivalent circuit. The solution of the secondary equivalent circuit is dependent on the solution of the primary equivalent circuit since the induced secondary voltage is equal to $-j\omega Mi_p$.

Primary Equivalent Circuits. Two equivalent primary circuits for air-core transformers are shown in Fig. 13.2.

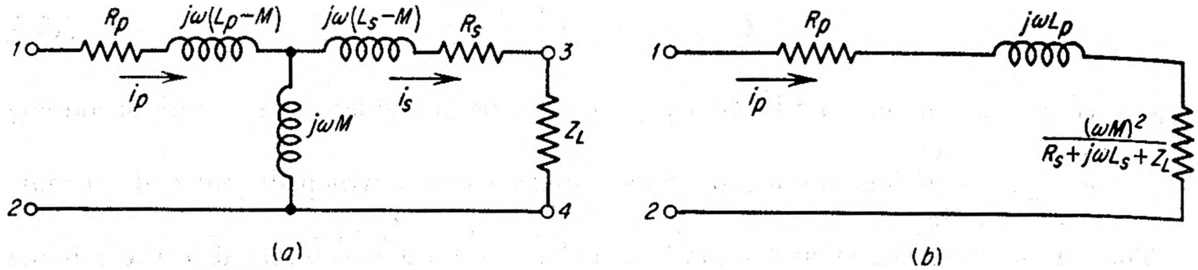


FIG. 13.2. Equivalent circuits for determining primary current.

Secondary Equivalent Circuit. The equivalent secondary circuit for a circuit of the type shown in Fig. 13.1 is shown in Fig. 13.3. The secondary current is that current which flows if the secondary induced voltage $-j\omega Mi_p$ is placed in series with the secondary circuit.

13.1e. Primary and Secondary Circuits Each Tuned to the Same Frequency. For inductively coupled circuits tuned to the same frequency, Fig. 13.4 illustrates how the primary current and the secondary voltage vary as a function of frequency and the coefficient of coupling. The degrees of coupling most commonly referred to are:

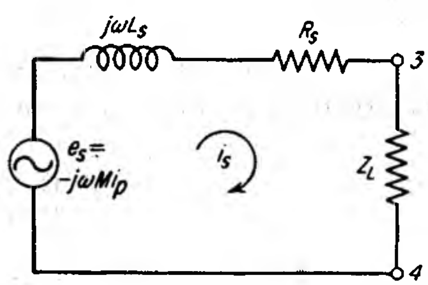


FIG. 13.3. Equivalent secondary for the circuit shown in Fig. 13.1.

1. Undercoupling
2. Critical coupling
3. Transitional coupling
4. Overcoupling

13.1f. Primary and Secondary Circuits Tuned to Different Frequencies. If two inductively coupled circuits of equal Q are tuned to slightly different frequencies, the response curve is similar in shape but smaller in amplitude than that obtained by overcoupling the same two circuits when tuned to the same frequency. The amplitudes of the two peaks in the secondary response curve are a function of the circuit

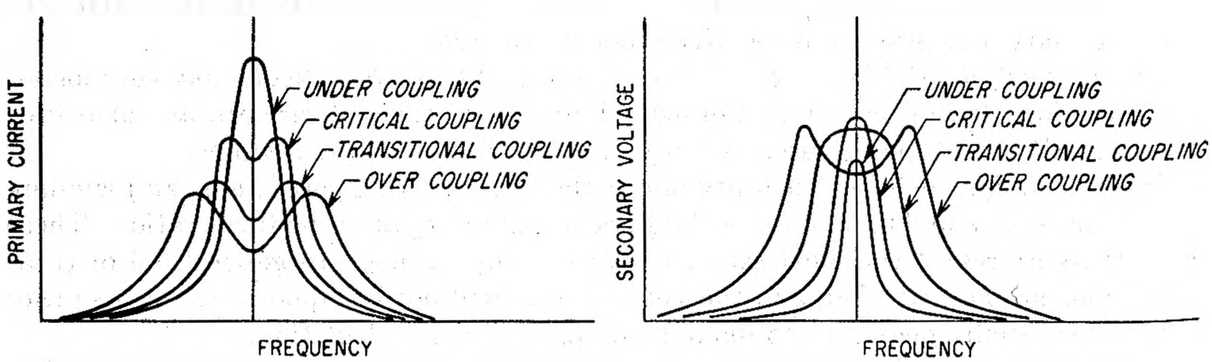


FIG. 13.4. Primary current and secondary voltage as a function of the coefficient of coupling when the primary and secondary are tuned to the same frequency.

Q's. If Q_p does not equal Q_s the secondary response curve will be unsymmetrical. If the detuning between the primary and secondary becomes very large, the primary current will have only a single peak even though the secondary response curve has double peaks.

13.1g. T Sections as Equivalents for Inductively Coupled Circuits. There are occasions when it is advantageous to replace inductively coupled circuits with equivalent T sections. One particular example is the case where the inductances of the windings are to be adjusted to resonate with fixed primary and secondary capacities. The coupled circuit in Fig. 13.5*a* can be replaced mathematically and sometimes physically by the equivalent *T* section of Fig. 13.5*b*. Frequently it is found, in the calculation of the series arms of the T section, that either $L_p - M$ or $L_s - M$ is a negative inductance. This is the case when k is greater than either the quantity $\sqrt{L_p/L_s}$ or $\sqrt{L_s/L_p}$, respectively. Negative inductance presents no calculation difficulties; however, it does present a problem in a physical network. If the physical T is to replace the coupled circuit at only one frequency, the negative inductance can be represented by capacitance. If a T section is to physically replace a coupled circuit for a

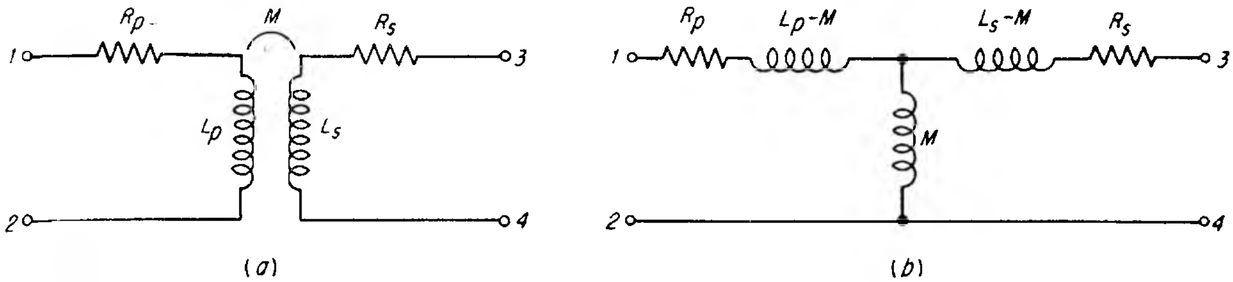


FIG. 13.5. Inductively coupled circuit and equivalent T section.

wide band of frequencies, the quantities $L_p - M$ and $L_s - M$ must have positive signs. This can be assured if the mutual inductance of the coupled circuit is made less than either the primary or secondary inductance.

In some instances there may be an advantage in converting the T network to a π network. If the T network has no negative inductances, the equivalent π network will also have no negative inductances.

It is important to note that, for an inductively coupled circuit terminated with a noncapacitive load, *it is not possible to realize a voltage gain if the mutual inductance is equal to or less than the primary inductance.* Therefore, where T sections are used to simulate inductively coupled circuits over a band of frequencies, there will be no voltage gain if the secondary load is not capacitive. For operation at *any specific frequency,* the unity-voltage gain limitation for a T section with a resistive load can be eliminated by making $L_p - M$ sufficiently negative and replacing the quantity with a capacitor.

13.1h. Gain-bandwidth Factor. A useful term in the evaluation of coupled circuits is *gain-bandwidth factor.* This is the ratio of the gain-bandwidth product of a given circuit to the gain-bandwidth product of a single-tuned circuit which has the same circuit capacitance as the sum of the primary and secondary capacitances of the coupled circuit. A single-tuned circuit is defined as the parallel combination of a single inductance, capacitance, and resistance.

Example 13.1

For the circuit shown in Fig. 13.6, determine the primary current and the voltage across terminals 3 and 4.

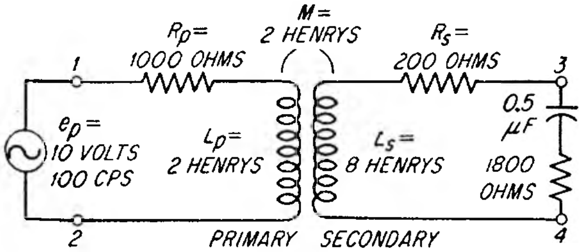


FIG. 13.6. Inductively coupled circuit.

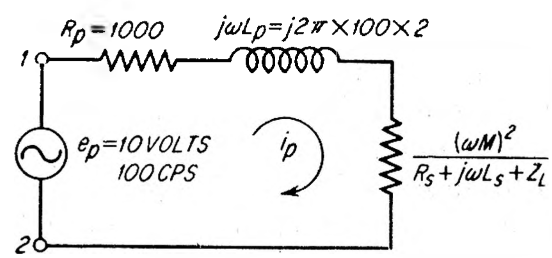


FIG. 13.7. Equivalent primary circuit for the circuit shown in Fig. 13.6.

Solution

1. Draw the equivalent primary circuit.

The equivalent primary circuit is shown in Fig. 13.7 and is of the same type as that shown in Fig. 13.2*b*.

2. Determine the primary current i_p.

$$i_p = \frac{\text{primary voltage}}{\text{primary input impedance}}$$

$$= \frac{e_p}{R_p + j\omega L_p + \dfrac{(\omega M)^2}{R_s + j\omega L_s + Z_L}}$$

$$= \frac{10}{1{,}000 + j2\pi \times 100 \times 2 + \dfrac{(2\pi \times 100 \times 2)^2}{200 + j2\pi \times 100 \times 8 + 1{,}800 - \dfrac{j}{2\pi \times 100 \times 0.5 \times 10^{-6}}}}$$

$$= \frac{10}{1{,}000 + j1{,}256 + 428 - j393} = \frac{10}{1{,}668\underline{/+31.2^\circ}} = 0.006\underline{/-31.2^\circ} \text{ amp}$$

or

$$= 0.0051 - j0.0031 \text{ amp}$$

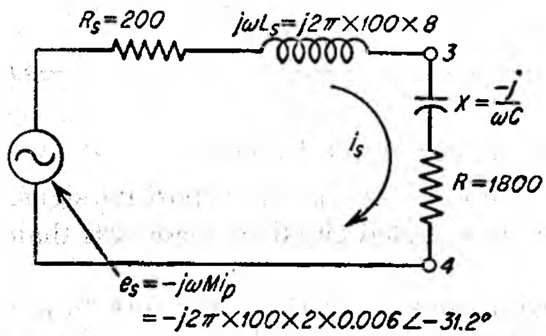


FIG. 13.8. Equivalent secondary circuit for the circuit shown in Fig. 13.6.

3. Draw the equivalent secondary circuit as shown in Fig. 13.8.

4. Determine the secondary voltage e_L across terminals 3 and 4.

$$e_L = i_s Z_L$$

$$i_s = \frac{e_s}{Z_s}$$

$$\therefore e_L = \frac{e_s Z_L}{Z_s}$$

where i_s = secondary current
e_s = secondary induced voltage
Z_s = secondary circuit impedance
Z_L = impedance between terminals 3 and 4
e_L = voltage across terminals 3 and 4

$$e_L = \frac{-j\omega M i_p Z_L}{R_s + j\omega L_s + Z_L}$$

$$= \frac{(-j2\pi \times 100 \times 2)(0.0051 - j0.0031)\left(1{,}800 - \dfrac{j}{2\pi \times 100 \times 0.5 \times 10^{-6}}\right)}{200 + j2\pi \times 100 \times 8 + 1{,}800 - \dfrac{j}{2\pi \times 100 \times 0.5 \times 10^{-6}}}$$

$$= \frac{7.54\underline{/-121.2^\circ} \times 3{,}657\underline{/-60.5^\circ}}{2{,}716\underline{/+42.6^\circ}}$$

$$= 10.15\underline{/+135.7^\circ} \text{ volts}$$

13.2. Undercoupling. Two circuits which are inductively coupled and tuned to the frequency of the applied signal are said to be *undercoupled* if their orientation is such that increased coupling will cause an increase in the secondary voltage. The curves in Fig. 13.4*b* illustrate how the bandwidth characteristics vary as the degree of coupling is changed. The gain of an undercoupled circuit can be expressed as some value relative to the maximum possible value of gain which occurs at critical coupling (see Sec. 13.3).

The curve in Fig. 13.9 is a plot of the secondary voltage in an undercoupled circuit relative to the voltage at critical coupling. This is given as a function of the ratio between the actual coupling coefficient k to the critical coupling coefficient k_c. If the value of k is much less than critical, the shape of the secondary voltage response curve approximates the product of the response curves of two circuits having Q's which are equal to the primary and secondary circuit Q's, respectively. As the coupling coefficient is increased, the bandwidth of the secondary voltage response curve also increases.

13.3. Critical Coupling. Two circuits which are inductively coupled and tuned to the frequency of the applied signal are said to be *critically coupled* if they have been oriented so as to obtain the maximum possible secondary voltage.

The circuits in Figs. 13.10 and 13.11 illustrate the two most common forms of coupled circuit configurations. In Fig. 13.10 the circuit Q's are directly proportional to the size of the shunting resistors, provided that the Q's of the windings are much higher than the Q's of the circuits when loaded by the shunting resistors. This

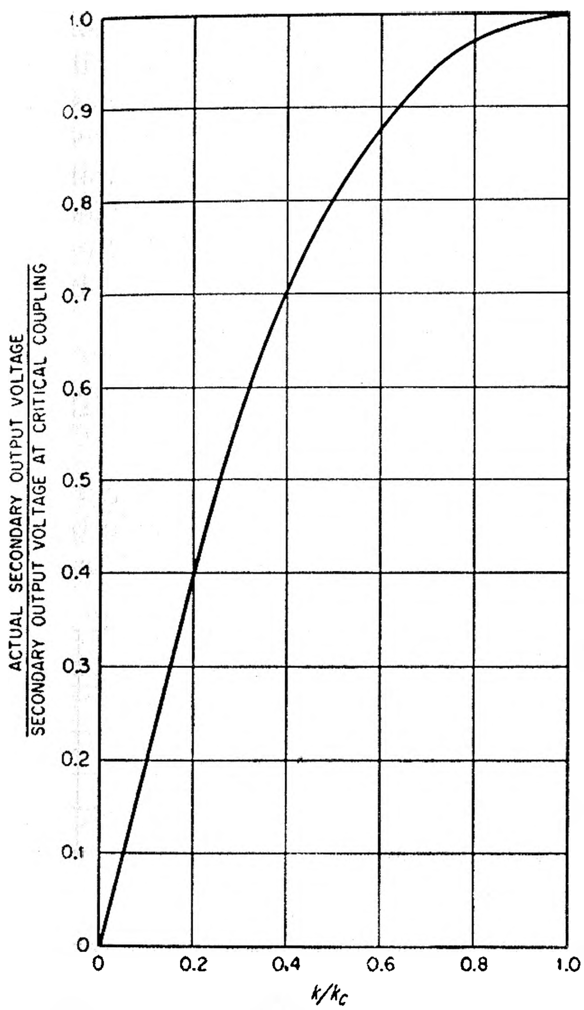
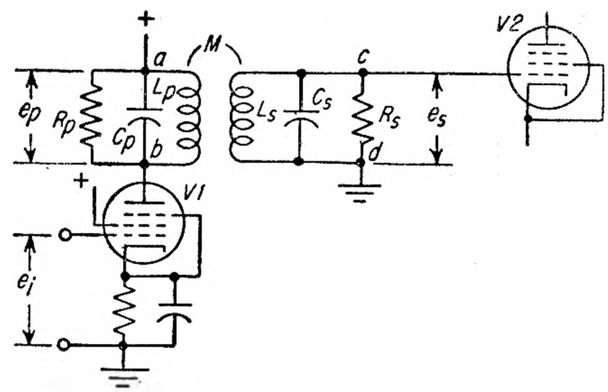


FIG. 13.9. A plot of the relative secondary output voltage as a function of the ratio between the actual coupling coefficient and the critical coupling coefficient for two circuits tuned to the same frequency and inductively coupled together.



$M = k_c \sqrt{L_p L_s}$

$Q_p = 2\pi f_r C_p R_p$

$Q_s = 2\pi f_r C_s R_s$

$\frac{e_s}{e_p} = \sqrt{\frac{R_s}{R_p}}$ (AT f_r)

$f_r = \frac{1}{2\pi\sqrt{L_p C_p}} = \frac{1}{2\pi\sqrt{L_s C_s}}$

$Z_i = \frac{R_p}{2}$ (AT FREQ OF MAX GAIN)

= INPUT IMPEDANCE AT TERMINALS a & b AT f_r

$Z_{12} = \frac{\sqrt{R_p R_s}}{2}$

= TRANSFER IMPEDANCE FROM TERMINALS a-b TO c-d AT FREQUENCY OF MAXIMUM GAIN

$A = \frac{e_s}{e_i} \simeq g_m Z_{12}$

FIG. 13.10. Shunt-loaded circuit and equations for critical coupling.

NOTE: When referring to Figs. 13.12 through 13.15, Q_p and Q_s can be identified as Q_1 and Q_2 or Q_2 and Q_1, respectively.

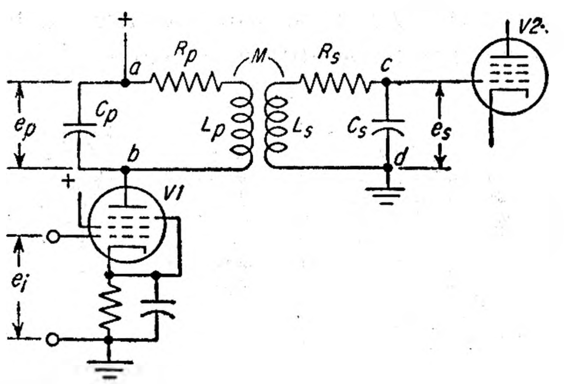
circuit is usually used when low-Q circuits are desired. If high-Q circuits are employed, the shunting resistors are omitted, resulting in the circuit shown in Fig. 13.11. If such is the case, the circuit Q's are determined entirely by the primary and secondary windings.

The general circuit equations for the two circuits with critical coupling are quite complex; hence, the circuit data have been plotted in Figs. 13.12 to 13.15. There is a limitation in the application of these curves since they *are not valid* for the circuit shown in Fig. 13.11 if either circuit Q is less than approximately 10. The data plotted in Figs. 13.12 to 13.15 are based on a constant current source, which is approached with most pentode-type tubes. In all the figures, Q_p and Q_s can be identified as either Q_1 and Q_2 or Q_2 and Q_1, respectively.

13.3a. Gain-bandwidth Factor, Critically Coupled Circuit. This is a function of the Q ratio and the relative values of the primary and secondary circuit capacitances. The gain-bandwidth factor is greatest when $Q_p = Q_s$ and when C_p is either much larger or much smaller than C_s. Figure 13.12 is a plot of the gain-bandwidth factor for the case where the primary circuit capacitance C_p is equal to the secondary circuit capacitance C_s. Under these conditions, the Q ratio must be less than 4.5 to realize any improvement in the gain-bandwidth product over a single-tuned stage provided that the primary and secondary circuit capacitances are equal. If the capacitances are unequal, the gain-bandwidth factor established from Fig. 13.12 must be multiplied by the factor K, as determined by Eq. (13.10) to obtain the actual gain-bandwidth factor.

$$K = \frac{C_p + C_s}{2\sqrt{C_pC_s}} \tag{13.10}$$

It is therefore possible, with Q ratios larger than 4.5, to achieve gain-bandwidth products that are greater than that achievable with a single-tuned circuit provided the improvement due to the term K is sufficiently large.



$$M = k_c\sqrt{L_pL_s}$$

$$Q_p = \frac{1}{2\pi f_r C_p R_p} = \frac{2\pi f_r L_p}{R_p}$$

$$Q_s = \frac{1}{2\pi f_r C_s R_s} = \frac{2\pi f_r L_s}{R_s}$$

$$\frac{e_s}{e_p} = \frac{Q_s}{Q_p}\sqrt{\frac{R_s}{R_p}} \quad (\text{AT } f_r)$$

$$f_r = \frac{1}{2\pi\sqrt{L_pC_p}} = \frac{1}{2\pi\sqrt{L_sC_s}}$$

$$Z_i = \frac{Q_p\pi f_r L_p}{2} = \frac{Q_p^2 R_p}{2} \quad (\text{IF } Q_p \geq 3)$$

= INPUT IMPEDANCE AT TERMINALS a & b AT f_r

$$Z_{12} = \frac{Q_pQ_s\sqrt{R_pR_s}}{2} = \frac{1}{4\pi f_r}\sqrt{\frac{Q_p\,Q_s}{C_p\,C_s}} \quad (\text{if } Q_p \geq 3)$$

= TRANSFER IMPEDANCE FROM TERMINALS a-b TO c-d AT f_r

$$A = \frac{e_s}{e_i} \simeq g_m Z_{12}$$

FIG. 13.11. Series-loaded circuit and equations for critical coupling.

NOTE: When referring to Figs. 13.12 through 13.15, Q_p and Q_s can be identified as Q_1 and Q_2 or Q_2 and Q_1, respectively. Figures 13.12 through 13.15 *are not valid* for this circuit if either Q_p or Q_s is less than approximately 10.

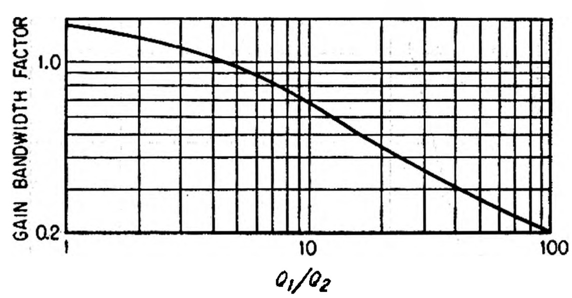


FIG. 13.12. Gain-bandwidth factor versus Q ratio for a critically coupled circuit in which $C_p = C_s$. If C_p does not equal C_s, the value as read from the curve must be corrected by the factor K given by Eq. (13.10).

13.3b. Gain Ratio, Critically Coupled Circuit. The *gain ratio* G_r is the ratio of the voltage gain of a critically coupled stage to the voltage gain of a single-tuned stage having the same total circuit capacitance.

$$G_r = \frac{\sqrt{Q_pQ_s}}{Q_o}\,\frac{C_p + C_s}{2\sqrt{C_pC_s}} \tag{13.11}$$

where Q_o = Q of single-tuned circuit
$C_p + C_s$ = total circuit capacitance of single-tuned circuit

The gain ratio is useful in comparing the voltage gain of a critically coupled stage to the voltage gain of a single-tuned stage without regard to relative bandwidths. If the critically coupled stage is adjusted to have the same bandwidth as the single-tuned stage, the gain ratio will be equal to the gain bandwidth factor. If maximum voltage gain is the prime objective and bandwidth is of no importance, there is ordinarily no advantage in using a critically coupled double-tuned circuit over a single-tuned circuit unless C_p and C_s are unequal since the maximum achievable value of $\sqrt{Q_pQ_s}$ in Eq. (13.11) will usually be no greater than the maximum value of Q_o.

13.3c. Low-Q Critically Coupled Circuits. The curves shown in Fig. 13.13 are an aid in the design of low-Q critically coupled circuits. The ratio of the primary and secondary resonant frequency f_r to the frequency f_{ac} which is the arithmetic center of the passband is plotted versus Q_2.

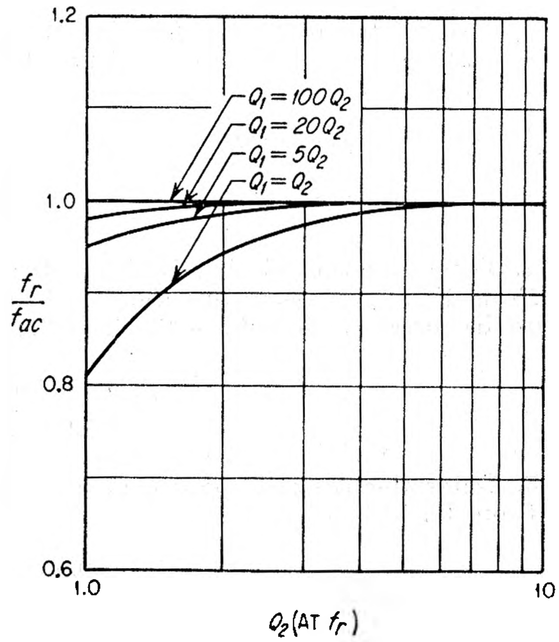


FIG. 13.13. $\frac{f_r}{f_{ac}}$ versus Q_2 for different Q ratios in critically coupled circuits.

13.3d. Fractional Bandwidth, Critically Coupled Circuit. Figure 13.14 is a plot of the fractional bandwidth, i.e., the ratio of bandwidth β to the arithmetic-center frequency f_{ac}, versus Q_2 for different Q ratios. Bandwidth is not affected by the ratio of the primary and secondary circuit capacitances.

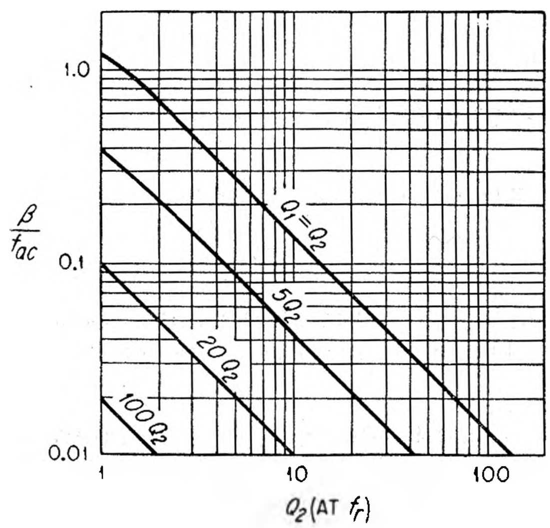


FIG. 13.14. Fractional bandwidth versus Q_2 for different Q ratios in critically coupled circuits.

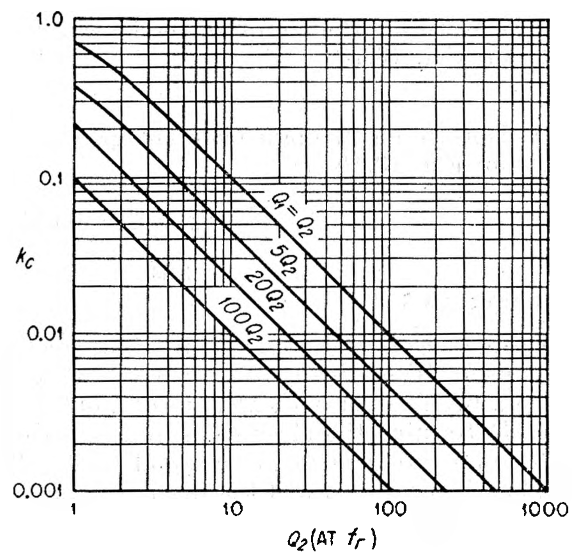


FIG. 13.15. Critical coupling coefficient versus Q_2 for different Q ratios in critically coupled circuits.

13.3e. Coupling Coefficient, Critically Coupled Circuit. Figure 13.15 is a plot of the critical coupling coefficient k_c versus Q_2 for different Q ratios. Equation (13.12) is an approximation which very nearly represents the data plotted in Fig. 13.15 provided both Q_1 and Q_2 are greater than approximately 5.

$$k_c = \frac{1}{\sqrt{Q_1Q_2}} \tag{13.12}$$

Example 13.2

Determine the operating characteristics of a critically coupled circuit of the type shown in Fig. 13.10 if

$R_p = 2{,}000$ ohms $C_p = 2.5C_s$
$R_s = 1{,}000$ ohms $f_r = 1.5$ Mc
$Q_p = 5$ $V1 =$ 6AK5 tube; $g_m = 5{,}000$ μmhos, $r_p = 0.3$ megohm
$Q_s = 1$

1. Determine the Q ratio.

$$\frac{Q_p}{Q_s} = 5$$

2. Determine the gain-bandwidth factor.

From Fig. 13.12 determine the gain-bandwidth factor for $Q_1/Q_2 = 5$. The figure 0.95 must be corrected since $C_p = 2.5C_s$. Therefore the gain-bandwidth factor is equal to

$$0.95 \times \frac{2.5C_s + C_s}{2\sqrt{2.5C_s \times C_s}} = 1.05$$

3. Determine the arithmetic center f_{ac} of the secondary response curve.

From Fig. 13.13

$$\frac{f_r}{f_{ac}} = 0.95$$

$$f_{ac} = \frac{f_r}{0.95} = \frac{1.5}{0.95} = 1.58 \text{ Mc}$$

4. Determine the bandwidth β of the secondary response curve.

From Fig. 13.14

$$\frac{\beta}{f_{ac}} = 0.39$$

$$\beta = 0.39 \times 1.58 = 0.616 \text{ Mc, or } 616 \text{ kc}$$

5. Determine the coupling coefficient k_c.

From Fig. 13.15

$$k_c = 0.38$$

6. Determine the gain from the grid of $V1$ to the grid of $V2$.

From Fig. 13.10

$$\frac{e_s}{e_i} \simeq g_m Z_{12} = 0.005 \times \sqrt{2{,}000 \times 1{,}000}/2$$

$$\simeq 3.54$$

Example 13.3

Design a critically coupled circuit of the type shown in Fig. 13.11 which has the secondary response arithmetically centered at 465 kc and a bandwidth of 13 kc. Assume the tube to be a 6AK5 operating at a g_m of 5,000 μmhos.

1. Determine the circuit Q's and the Q ratio.

Assume the Q ratio to be equal to 1. The fractional bandwidth is equal to 13/465, or 0.028. From Fig. 13.14 the desired value of the primary and secondary Q's is equal to 50.

2. Determine the value of the primary and secondary circuit capacitors.

To avoid any appreciable change in the total primary and secondary circuit capacitances which might be caused either by interchanging tubes or by changes in the input capacitance to $V2$, it usually is desirable to make the total circuit capacitances large compared to the expected variations. The nominal output and input capacitances of a 6AK5 tube are 2.8 and 4 μμf, respectively; hence it is relatively safe to assume that if both C_p and C_s were arbitrarily made equal to 50 μμf, the variations in tube capacitances would have little effect on the total. Therefore let

$$C_p = C_s = 50 \text{ μμf}$$

The smaller C_p and C_s, the larger will be the gain as determined in step 3. Consequently, C_p and C_s should be made no larger than necessary to prevent excessive detuning due to variations in circuit capacitances.

3. Determine the gain from the grid or $V1$ to the grid of $V2$.
From Fig. 13.11

$$\frac{e_s}{e_i} \simeq g_m Z_{12}$$

$$\simeq \frac{g_m}{4\pi f_r} \sqrt{\frac{Q_p Q_s}{C_p C_s}}$$

$$\simeq \frac{0.005}{4\pi \times 465 \times 10^3} \sqrt{\frac{(50)^2}{(50 \times 10^{-12})^2}}$$

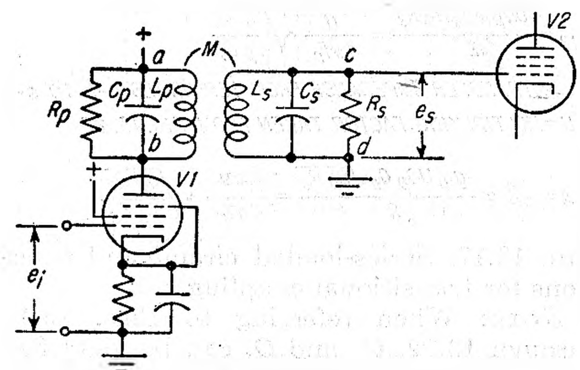
$$\simeq 856$$

4. Determine the critical coupling coefficient k_c.
From Fig. 13.15

$$k_c = 0.02$$

13.4. Transitional Coupling. Two inductively coupled circuits are said to be *transitionally coupled* if they are coupled to give the flattest secondary response curve possible. Transitional coupling will provide the widest passband without double peaks. Increased coupling will cause the midband portion of the selectivity curve to dip, and the circuit is then said to be overcoupled.

Transitional coupling is frequently referred to as optimum coupling. The gain-bandwidth product of a transitionally coupled circuit is always greater than that of a critically coupled circuit except in the case where the primary and secondary Q's are equal. In the case of equal-Q primary and secondary circuits, transitional coupling is identical to critical coupling. The improvement in the gain-bandwidth product over that of a critically coupled circuit is obtained at the expense of a loss in gain. The loss in gain is accompanied by an even larger increase in bandwidth, which accounts for the larger gain-bandwidth product. Unequal Q circuits which are transitionally coupled may develop pronounced asymmetry in gain with slight mistuning.



$$M = k_t \sqrt{L_p L_s}$$

$$Q_p = 2\pi f_p C_p R_p$$

$$Q_s = 2\pi f_s C_s R_s$$

$$f_p = \frac{1}{2\pi\sqrt{L_p C_p}}$$

$$f_s = \frac{1}{2\pi\sqrt{L_s C_s}}$$

$$Z_{12} = \frac{U\sqrt{R_p R_s}}{2}$$

= TRANSFER IMPEDANCE FROM TERMINALS a-b TO c-d

U = MULTIPLYING FACTOR TAKEN FROM FIG. 13.22

$$A = \frac{e_s}{e_i} \simeq \frac{g_m U \sqrt{R_p R_s}}{2}$$

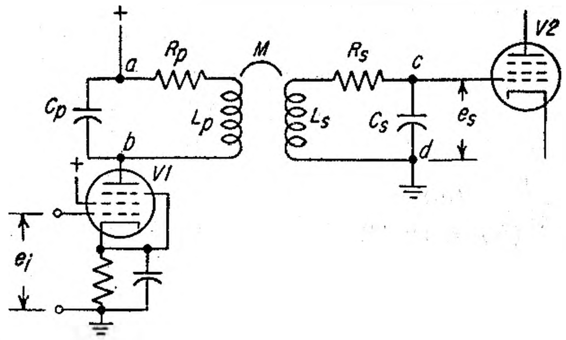
FIG. 13.16. Shunt-loaded circuit and equations for transitional coupling.

NOTE: When referring to Figs. 13.18 through 13.22, Q_p and Q_s can be identified as Q_1 and Q_2 or Q_2 and Q_1, respectively.

In Fig. 13.16, the circuit Q's are directly proportional to the size of the shunting resistors provided that the Q's of the windings are much higher than the Q's of the circuits when loaded by the shunting resistors. This circuit is usually used when low-Q circuits are desired.

In high-Q circuits, the shunting resistors are omitted, resulting in the circuit shown in Fig. 13.17. If such is the case, the circuit Q's are determined entirely by the Q's of the primary and secondary windings.

The general circuit equations for the two circuits with transitional coupling are quite complex; hence, the circuit data have been plotted in Figs. 13.18 to 13.22. There is a limitation in the application of these curves in that they *are not valid* for the circuit shown in Fig. 13.17 if either circuit Q is less than approximately 10. The data plotted in Figs. 13.18 to 13.22 are based on a constant current source which is approached with most pentode-type tubes.



$$M = k_t \sqrt{L_p L_s}$$

$$Q_p = \frac{1}{2\pi f_r C_p R_p} = \frac{2\pi f_r L_p}{R_p}$$

$$Q_s = \frac{1}{2\pi f_r C_s R_s} = \frac{2\pi f_r L_s}{R_s}$$

$$f_r = \frac{1}{2\pi \sqrt{L_p C_p}} = \frac{1}{2\pi \sqrt{L_s C_s}} \quad (\text{FOR } Q_p \text{ \& } Q_s \geq 10)$$

$$Z_{12} = \frac{U Q_p Q_s \sqrt{R_p R_s}}{2} = \frac{U}{4\pi f_r} \sqrt{\frac{Q_p Q_s}{C_p C_s}}$$

= TRANSFER IMPEDANCE FROM TERMINALS a–b TO c–d

U = MULTIPLYING FACTOR TAKEN FROM FIG. 13.22

$$A = \frac{e_s}{e_i} \simeq \frac{g_m U Q_p Q_s \sqrt{R_p R_s}}{2} = \frac{g_m U}{4\pi f_r} \sqrt{\frac{Q_p Q_s}{C_p C_s}}$$

FIG. 13.17. Series-loaded circuit and equations for transitional coupling.

NOTE: When referring to Figs. 13.18 through 13.22, Q_p and Q_s can be identified as Q_1 and Q_2 or Q_2 and Q_1, respectively. Figures 13.18 through 13.22 *are not valid* for this circuit if either Q_p or Q_s is less than approximately 10.

13.4a. Gain-bandwidth Factor, Transitionally Coupled Circuit. The gain-bandwidth factor is between 1.41 and 2 for the case of equal primary and secondary circuit capacitances. If the capacitances are unequal, the gain-bandwidth factor as determined from Fig. 13.18 should be multiplied by $(C_p + C_s)/(2\sqrt{C_p C_s})$.

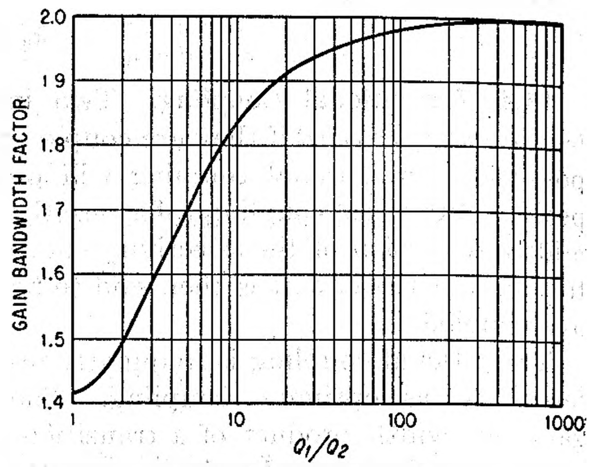


FIG. 13.18. Gain-bandwidth factor versus Q ratio for a transitionally coupled double-tuned circuit in which $Q_2 \geq 3$. If C_p does not equal C_s, the gain-bandwidth factor must be multiplied by $(C_p + C_s)/2\,(\sqrt{C_p C_s})$

13.4b. Bandwidth of Cascaded Transitionally Coupled Stages. The bandwidth β_n of n identical transitionally coupled stages which are cascaded and synchronously tuned[1] is given by Eq. (13.13).

$$\beta_n = \beta \sqrt[4]{2^{1/n} - 1} \tag{13.13}$$

$$\beta_n \simeq \frac{\beta}{1.1 \sqrt[4]{n}} \tag{13.14}$$

where β_n = bandwidth between −3-db frequencies for n stages
n = number of stages
β = bandwidth between −3-db frequencies for one stage

13.4c. Gain Ratio, Transitionally Coupled Circuit. Without regard to relative bandwidths, the *gain ratio* G_r is the ratio of the voltage gain of a transitionally coupled stage to the voltage gain of a single-tuned stage having the same total circuit capacitance.

$$G_r = \frac{\sqrt{Q_p Q_s}}{Q_o} \frac{C_p + C_s}{2\sqrt{C_p C_s}} U \tag{13.15}$$

[1] A cascaded synchronously tuned amplifier is one in which successive stages are tuned to the same frequency.

where Q_o = Q of single-tuned circuit
$C_p + C_s$ = total circuit capacitance of single-tuned circuit

$$U = \frac{\text{gain at transitional coupling}}{\text{gain at critical coupling}} \quad \text{(see Fig. 13.22)}$$

If the transitionally coupled stage is adjusted to have the same bandwidth as the single-tuned stage, the gain ratio will be equal to the gain bandwidth factor.

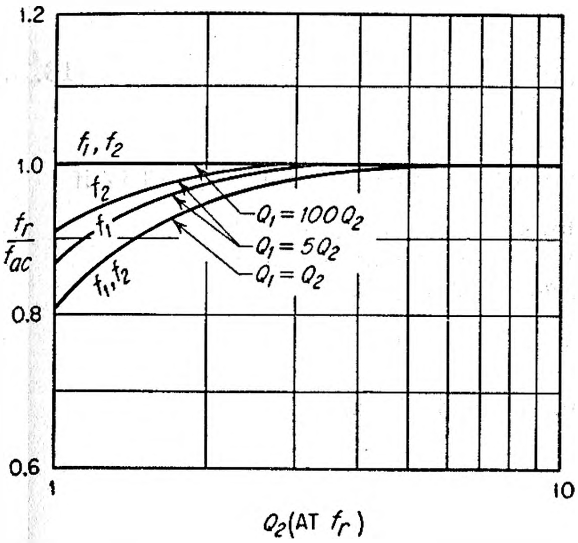


FIG. 13.19. Ratio of circuit resonant frequencies to the arithmetic center of the secondary response curve for different Q ratios versus Q_2 for transitionally coupled circuits.

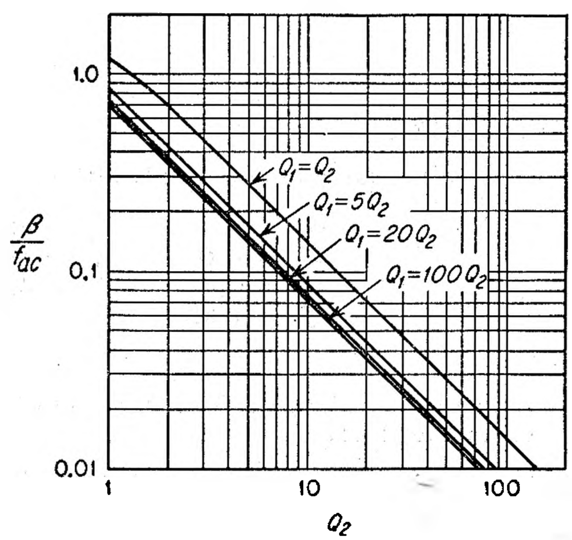


FIG. 13.20. Fractional bandwidth versus Q_2 for different Q ratios in transitionally coupled circuits.

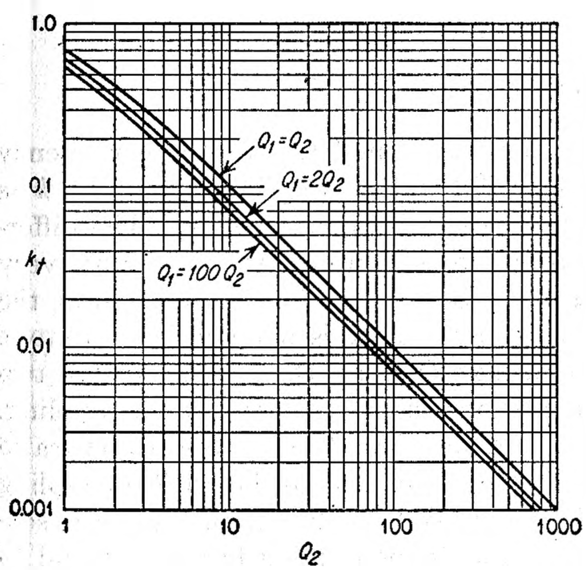


FIG. 13.21. Transitional-coupling coefficient versus Q_2 for different Q ratios in transitionally coupled circuits.

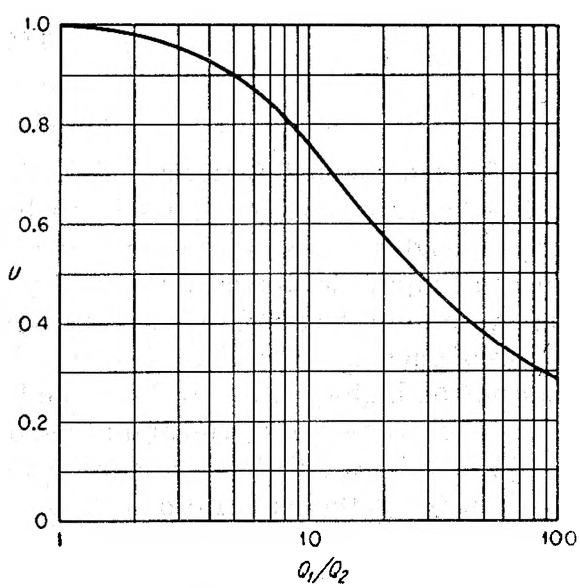


FIG. 13.22. Ratio of the gain in a transitionally coupled circuit to the gain in a critically coupled circuit versus circuit Q ratio.

13.4d. Low-Q Transitionally Coupled Circuits. In low-Q circuits, the primary and secondary resonant frequencies f_p and f_s may differ considerably from the arithmetic center f_{ac} of the passband. In most design problems f_{ac} is specified, and consequently f_p and f_s must be determined. Figure 13.19 is useful in determining f_p and f_s with respect to f_{ac}. The terms Q_2 and f_2 in the figure may refer to the Q and frequency of resonance of either the primary or secondary circuit depending on which has the lower Q.

13.4e. Fractional Bandwidth, Transitionally Coupled Circuit. Figure 13.20 shows the fractional bandwidth, i.e., the ratio of bandwidth β to the arithmetic center frequency f_{ac}, versus Q_2 for different Q ratios. Bandwidth is not affected by the ratio of the primary and secondary circuit capacitances.

13.4f. Coupling Coefficient, Transitionally Coupled Circuit. Figure 13.21 is a plot of k_t versus Q_2 for different Q ratios. Equation (13.16) very nearly represents the data plotted in Fig. 13.21 if both Q_1 and Q_2 are greater than approximately 5.

$$k_t = 0.707\sqrt{\frac{1}{Q_p^2} + \frac{1}{Q_s^2}} \qquad (13.16)$$

Example 13.4

Determine the gain and bandwidth of a circuit of the type shown in Fig. 13.17 if

$Q_p = 30$ $\quad R_p = 66.6$ ohms (a-c resistance of L_p)
$Q_s = 150$ $\quad R_s = 13.3$ ohms (a-c resistance of L_s)
$C_p = C_s$ $\quad V1 = $ 6AK5, $g_m = 5{,}000$ μmhos
$f_r = 1$ Mc

Solution

1. Determine the gain from the grid of $V1$ to the grid of $V2$.
From Fig. 13.17

$$\frac{e_s}{e_i} \simeq \frac{g_m U Q_p Q_s \sqrt{R_p R_s}}{2}$$

$$\simeq \frac{0.005 \times 0.895 \times 30 \times 150 \sqrt{66.6 \times 13.3}}{2}$$

$$\simeq 300$$

2. Determine the bandwidth.
From Fig. 13.20

$$\frac{\beta}{f_{ac}} = 0.029$$

$$\beta = 0.029 \times 10^6$$

$$\beta = 29 \times 10^3, \text{ or } 29 \text{ kc}$$

13.5. Overcoupled Circuits. Two circuits that are tuned to the same frequency and inductively coupled are said to be *overcoupled* if the coupling coefficient k is large enough to cause the secondary response curve to have two peaks. The difference in amplitude between the peaks is a function of the circuit Q's. With very high Q circuits the amplitudes of the peaks tend to be equal. For low-Q circuits, the low-frequency peak is larger and the high-frequency peak is smaller in amplitude than in the high-Q case. In both the low- and high-Q cases, the average of the two peaks is nearly equal to the amplitude of the response curve at transitional coupling.

13.5a. Midband Gain and Relative Gain of Peaks to Midband. For every ratio Q_p/Q_s or Q_s/Q_p there is a ratio of the actual coupling coefficient to the critical-coupling coefficient k/k_c above which increased coupling will result in double peaks in the secondary response curve. The value of k/k_c at which overcoupling begins is equal to the ratio of the transitional coupling coefficient to the critical coupling coefficient k_t/k_c. The intersection of the value of k/k_c which exists in a given circuit with the curve of Fig. 13.23 determines D where D is equal to the ratio of the midband gain (whether overcoupled or not) to the gain of the circuit with critical coupling. If the intersection of Q_1/Q_2 is at a higher point on the curve than the point of the intersection of k/k_c, the circuit is overcoupled and the ratio of the average amplitude of the peaks to the midband amplitude will be slightly less than the ratio of the two values of D read from Fig. 13.23 for Q_1/Q_2 and k/k_c. The applicable equations are

$$\text{Midband gain} = \text{gain of circuit at critical coupling} \times N \qquad (13.17)$$

$$\frac{\text{Average peak amplitude}}{\text{Midband amplitude}} \simeq \frac{RM}{N} \qquad (13.18)$$

where M = value of D at Q_1/Q_2 (see Fig. 13.23)
N = value of D at k/k_c (see Fig. 13.23)
R = a multiplying factor determined by Q ratio (see Fig. 13.24)

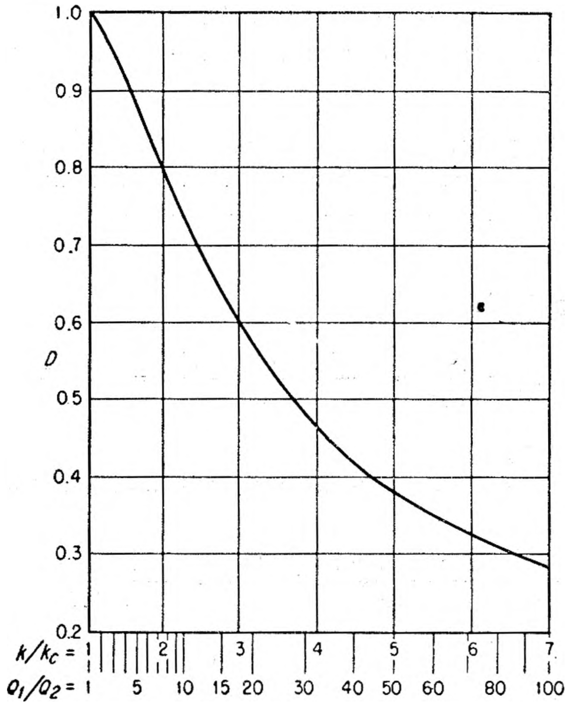


FIG. 13.23. Determination of circuit gain relative to the gain at critical coupling for ratios of k to k_c. The abscissa is calibrated in both k/k_c and Q_1/Q_2. The values of k/k_c associated with the values of Q_1/Q_2 are the maximum values at which double humps do not occur.

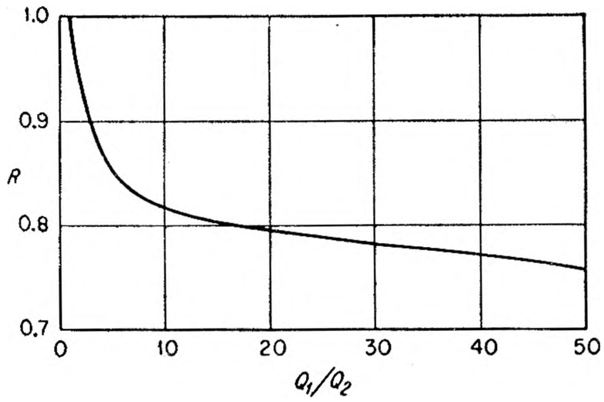


FIG. 13.24. Plot of the modifying factor R [see Eq. (13.18)] versus Q_1/Q_2.

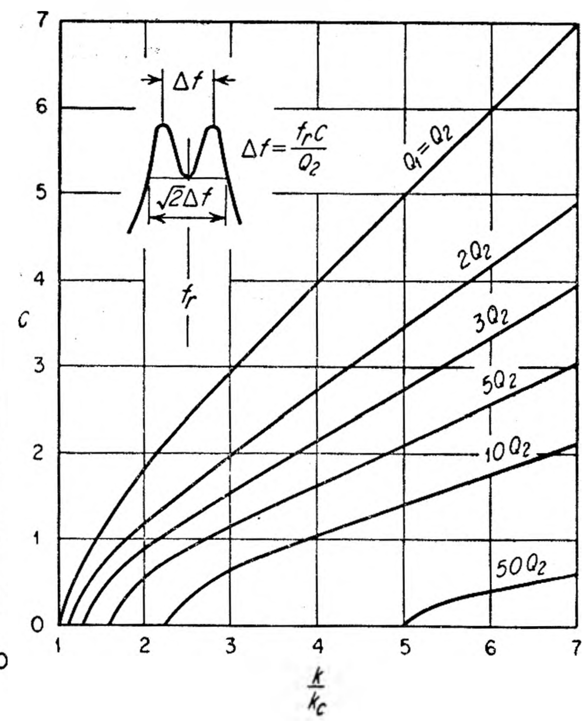


FIG. 13.25. Relationship between Δf, Q ratios, and k/k_c for overcoupled circuits which are tuned to the same frequency.

13.5b. Bandwidth of an Overcoupled Circuit. Because of the shape of the secondary response curve of an overcoupled circuit, it is not too helpful to refer to bandwidth at the −3-db points. Usually it is more desirable to define the secondary response curve as shown in Fig. 13.25.

13.5c. An Application of Overcoupled Circuits. A wide and fairly flat response curve is sometimes obtained from a two-stage amplifier in which the double-peak response of an overcoupled circuit is added to the single-peak response of a circuit that has transitional coupling or less. The overcoupled circuit is tuned so as to be symmetrical about the response curve of the other stage. To obtain this flat response, the circuit Q's, tuning and degree of coupling must be carefully chosen.

Example 13.5

In an overcoupled circuit where $k/k_c = 4$, $Q_2 = 10$, $Q_1 = 100$, and $f_r = 30$ Mc, determine Δf, ratio of the average peak amplitude to that at midband, and the gain at f_r based on the assumption that the gain at critical coupling is 30.

Solution

1. Determine Δf (Fig. 13.25)

$$\Delta f = \frac{f_r C}{Q_2} = \frac{30 \times 10^6 \times 1.05}{10} = 3.15 \text{ Mc}$$

where $C = 1.05$ (Fig. 13.25)

2. Determine the ratio of the average peak amplitude to that at midband.
From Eq. 13.18

$$\frac{\text{Average peak amplitude}}{\text{Midband amplitude}} \simeq \frac{RM}{N} = \frac{(0.82)(0.74)}{0.465} = 1.31$$

where $M = 0.74$ (from Fig. 13.23)
$N = 0.465$ (from Fig. 13.23)
$R = 0.82$ (from Fig. 13.24)

3. Determine the midband gain.
From Eq. 13.17

$$\begin{aligned}\text{Midband gain} &= \text{gain of circuit at critical coupling} \times N \\ &= 30 \times 0.465 \\ &= 13.95\end{aligned}$$