Regenerative Circuits With Real-Time Feedback Limiting

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10/31/2015

Abstract

The regenerative receiver is probably one of the most common TRF (Tuned Radio Frequency) AM radio receiver designs. While it is primarily intended to be operated below the onset of sustained local oscillations it is also frequently operated in "oscillating mode". Although mostly used to demodulate single sideband (SSB) transmissions the regenerative receiver in oscillating mode continues to be able to receive amplitude modulated signals[1] and there are often conflicting reports of whether oscillating mode provides better selectivity and sensitivity.

In a previous paper [1] a simple linear feedback model to study the basic behavior of regenerative receivers in non-oscillating mode for small input signals has been introduced. This model is now augmented by replacing the simple linear feedback device by a real-time limited feedback device to account for the fact that the maximum amount of feedback the device can deliver into the tuned circuit is limited. This will enable us to numerically study the circuit's behavior for large input signals in non-oscillating mode and also investigate it's oscillating mode.

For large input signals in non-oscillating mode, it is shown that the output signal is subject to an amplitude compression effect that also causes the large-signal bandwidth of the frequency response curve to be significantly larger then the small signal bandwidth. Next, the oscillating mode is studied, starting with the two basic phenomena injection locking and injection pulling. In particular, it is shown how injection pulling leads to beat frequencies much lower than the difference of the resonant frequency of the tank and the frequency of the input signal. It is shown that amplitude compression and a resulting increase in bandwidth also occur in oscillating mode. Most important, it is shown that when receiving amplitude modulated signals in oscillating mode, the carrier of the amplitude modulated signal is increased significantly. In combination with certain properties of the envelope detector, this "carrier boost" is able to significantly enhance the selectivity of the receiver without having an adverse effect on audio frequency bandwidth.

[1]More precisely: Double sideband modulation with carrier present.

Acknowledgement

The author wishes to thank Joseph L. Sousa and Prof. Dietmar Rudolph for reading the draft version of this paper and providing valuable comments and suggestions.

A Simple Bounded And Nonlinear Feedback Model

The generic linear feedback model that was introduced in [1] serves it's purpose quite well for studying the small signal behavior of a regenerative receiver in regular, i.e. non-oscillating mode. However, due to it's linear and unbounded nature, it can not be used to describe the large signal behavior or even the oscillating state of a regenerative receiver. In order to be able to do so, we need to augment this model and introduce bounds that will limit the amount of feedback provided to the lossy LC tank for large voltages. There are basically two ways to achieve this. First, the feedback device can be subject to some form of automatic gain control (AGC) with it's gain determined by the average voltage amplitude in the tank, taken over several cycles. This will typically be the case with vacuum tube regenerative circuits using grid leak detection where an increasing voltage amplitude in the tank will push the quiescent of the tube into regions with reduced transconductance. Mathematically, these AGC based feedback devices are difficult to treat since one will have to deal with combined integral and differential equations. The second option, and the method that we will pursue in this paper, is to introduce real time bounds into the feedback device by virtue of a bounded nonlinear feedback function.

Let us start with the schematic of our generic regenerative circuit as introduced in [1] and shown in figure 1.

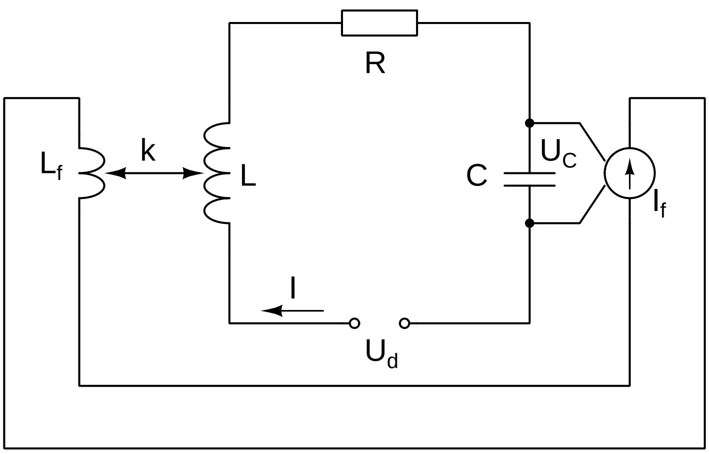

Figure 1: Generic regenerative circuit

Feedback into the main tank is delivered by a voltage controlled current source driving a feedback current I_{f} through the "tickler" coil L_{f} that is coupled inductively to the main tank coil L. The degree of inductive coupling is given by the coupling factor k, that is related to the mutual inductance M of the two coils [3] by $M = k\sqrt{LL_{\mathrm{f}}}$. The control voltage for the current source is the voltage U_C at the capacitor C of the main tank. We shall also refer to this voltage as the output voltage of the main tank. As usual, the losses in the main tank are modeled by a series loss resistance R and a driving voltage $U_{\mathrm{d}}(t)$ is applied [1].

Since the feedback current I_{f} needs to stay within reasonable limits, the function $I_{\mathrm{f}} = I_{\mathrm{f}}(U_C)$ needs to have upper and lower bounds. Also, since our goal is to augment our circuit model so that is contains the original linear feedback model as a small signal approximation we demand that for small voltages U_C, the function $I_{\mathrm{f}} = I_{\mathrm{f}}(U_C)$ be approximately linear. These requirements are nicely met by an arctangent function of the form

$$I_{\mathrm{f}}(U_C) = b_1 \cdot \arctan(b_2 U_C) \tag{1}$$

that is depicted in figure 2. Here, b_1 and b_2 are parameters whose physical meaning will become apparent soon.

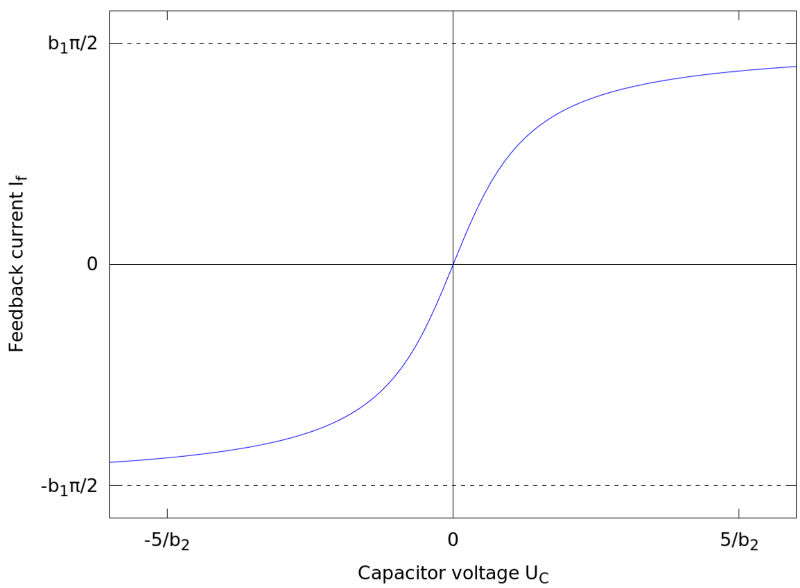

Figure 2: Feedback current $I_{\mathrm{f}} = I_{\mathrm{f}}(U_C)$

Since $-\pi/2 < \arctan(x) < \pi/2$, the bounds for the feedback current are $-b_1\pi/2 < I_{\mathrm{f}} < b_1\pi/2$ which immediately yields b_1 for any given feedback current limit. Also, from the Taylor series expansion of (1) around $U_C = 0$ it follows that

$I_\mathrm{f}(U_C) \approx b_1 b_2 \cdot U_C$ for sufficiently small values of U_C (small signal approximation). Obviously, the small signal transconductance of the feedback current source is $\beta = b_1 b_2$. Since b_1 is given by the feedback current limit, we can use b_2 to set the small signal transconductance of the feedback device. A practical example of such a real-time feedback device is a high input impedance differential amplifier followed by a voltage controlled current source.

Let us now derive the differential equation governing the circuit. We shall start with the feedback voltage U_f induced by the feedback coil into the main tank coil. Using the mutual inductance M between these two coils, we get [3]

$$U_\mathrm{f}(t) = -M \dot{I}_\mathrm{f}(t) \tag{2}$$

Also, applying Kirchhoff's loop rule [4] to the main tank leads to[2]

$$U_L(t) + U_R(t) + U_C(t) = U_\mathrm{d}(t) \tag{3}$$

Since the voltage $U_C(t)$ at the capacitor C and the current $I(t)$ in the main tank are connected by $I(t) = C\dot{U}_C(t)$, using U_f from equation (2) along with equation (1) in equation (3) and bearing in mind that $U_L(t) = L\dot{I}(t) + U_\mathrm{f}(t)$ then yields

$$LC\ddot{U}_C(t) - M\frac{d}{dt}\Big(b_1 \arctan\left(b_2 U_C(t)\right)\Big) + RC\dot{U}_C(t) + U_C(t) = U_\mathrm{d}(t)$$

from which we immediately obtain

$$\boxed{\frac{1}{\omega_0^2}\ddot{U}_C(t) + C \cdot \left(R - \frac{\frac{1}{C}Mb_1b_2}{1 + \left(b_2 U_C(t)\right)^2}\right)\dot{U}_C(t) + U_C(t) = U_\mathrm{d}(t)} \tag{4}$$

where $\omega_0 = 1/\sqrt{LC}$ is the resonant frequency of the main tank. Similar to the linear case [1], we can identify the virtual loss resistance in the above equation as

$$\boxed{\tilde{R} = R - \frac{\frac{1}{C}Mb_1b_2}{1 + \left(b_2 U_C(t)\right)^2}} \tag{5}$$

As expected, the virtual loss resistance in the nonlinear case is no longer a constant but depends explicitly on the output voltage $U_C(t)$ of the main tank. However, in the small signal case $U_C \ll 1/b_2$ the above nonlinear differential equation (4) is reduced to a linear differential equation with a constant virtual loss resistance of

$$\tilde{R} = R - \frac{1}{C}Mb_1b_2 \tag{6}$$

[2] bear in mind that the voltage loop must follow the (arbitrarily defined) positive direction of the current in the loop

In order to be able to obtain numeric solutions for the nonlinear differential equation (4) we need to choose reasonable values for the parameters involved. The parameters that will not require being changed in the course of this paper are set as follows: The capacitance in the main tank is set to $C = 100\text{pF}$ and a resonant frequency of $\omega_0 = 2\pi f_0$ with $f_0 = 1\text{MHz}$ is assumed[3]. The physical loss resistance is set to $R = 30\Omega$, leading to a physical Q-factor [5] of $Q \approx 53$ in the main tank. The mutual inductance between the feedback coil and the main tank coil shall be $M = 2\mu\text{H}$. The feedback parameter b_1 is set to $b_1 = 3\text{mA}$, limiting the feedback current according to equation (1) to $\pm 3\text{mA} \cdot \pi/2 \approx \pm 4.7\text{mA}$.

Non-oscillating Operation

Before advancing to the study of the oscillating mode let us spend some time to analyze the circuit in it's regular non-oscillating mode for signals that no longer qualify as "small". Let us take a closer look at the virtual loss resistance as given by equation (5) that in the nonlinear case depends explicitly on the output voltage U_C. In order to be safely inside the non-oscillating region we require $\tilde{R} > 0$ for all possible values of U_C. In this section, this is ensured by setting $b_2 = 0.4$, resulting in a small signal virtual loss resistance of $\tilde{R} = 6\Omega$ according to equation (6), respectively a virtual Q-factor of $Q \approx 265$. Obviously, $\tilde{R}$ reaches it's minimum for $U_C = 0$ and increases as $|U_C|$ increases. Hence, for small signals $U_C(t)$ the "average" value of $\tilde{R}$ and therefore the average damping is less then the average damping for larger signals.

Amplitude Compression

This behavior creates a real-time amplitude compression effect on the output voltage U_C, very similar to the use of a compressor in audio signal processing. In other words, if the amplitude of the driving voltage U_d is doubled, the output amplitude will be less than double. This behavior can also nicely be shown by numerically solving our differential equation (4) for different driving voltage amplitudes and plotting the output voltage amplitude versus the driving voltage amplitude as has been done in figure 3 for a driving frequency of 1MHz which is the resonant frequency of the main tank.

It immediately follows from equation (5) that amplitude compression is more pronounced in the case of stronger feedback ($|b_2|$ larger). Hence, a regenerative circuit having a main tank with a high physical Q-factor that requires less feedback to reach the desired virtual Q-factor will exhibit a more favorable large signal behavior.

[3]The equations governing the circuit are written in such a form that the inductance L never appears explicitly.

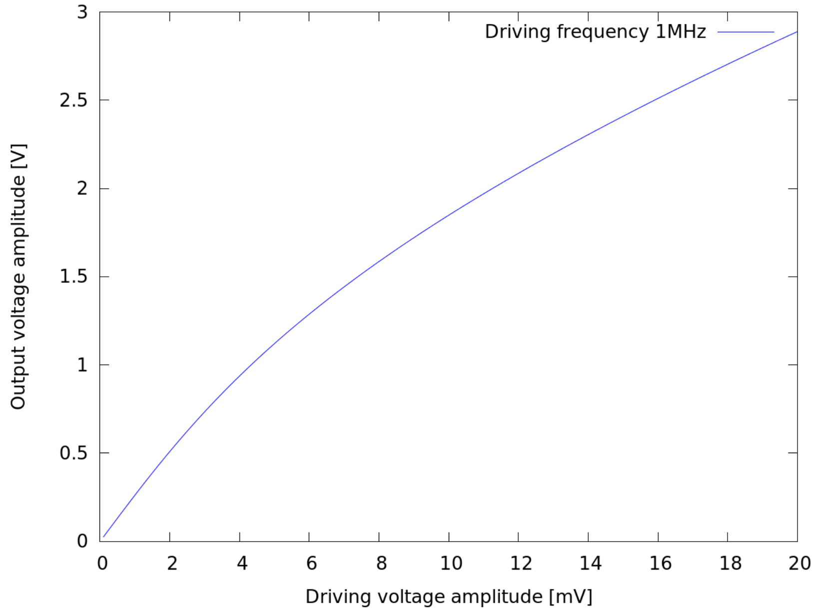

Figure 3: Amplitude compression

Bandwidth Widening

One immediate consequence that arises from the amplitude compression effect is an increased bandwidth of the frequency response curve of the main tank. This simply arises because the frequency response curve is subjected to compression on the output amplitude axis while leaving the frequency axis as it is, resulting in a less pronounced drop towards more off-resonant frequencies. To illustrate this behavior, the frequency response curve has been computed numerically from differential equation (4) for different driving voltage amplitudes and plotted in figure 4.

For comparison, the bandwidth for small input signals where the linear approximation (6) is valid is [5] $B = f_0/Q = 1\text{MHz}/265 = 3.77\text{kHz}$

The Oscillating State

Usually, the user of a regenerative receiver set is strongly reminded to avoid the oscillating state of the circuit. This is because the oscillations can cause interference in other nearby receivers. In particular, the beat between the frequency of the oscillator and a neighboring carrier of a radio station will be heard as the familiar whistle tone not only on the oscillating receiver but on nearby receivers as well.

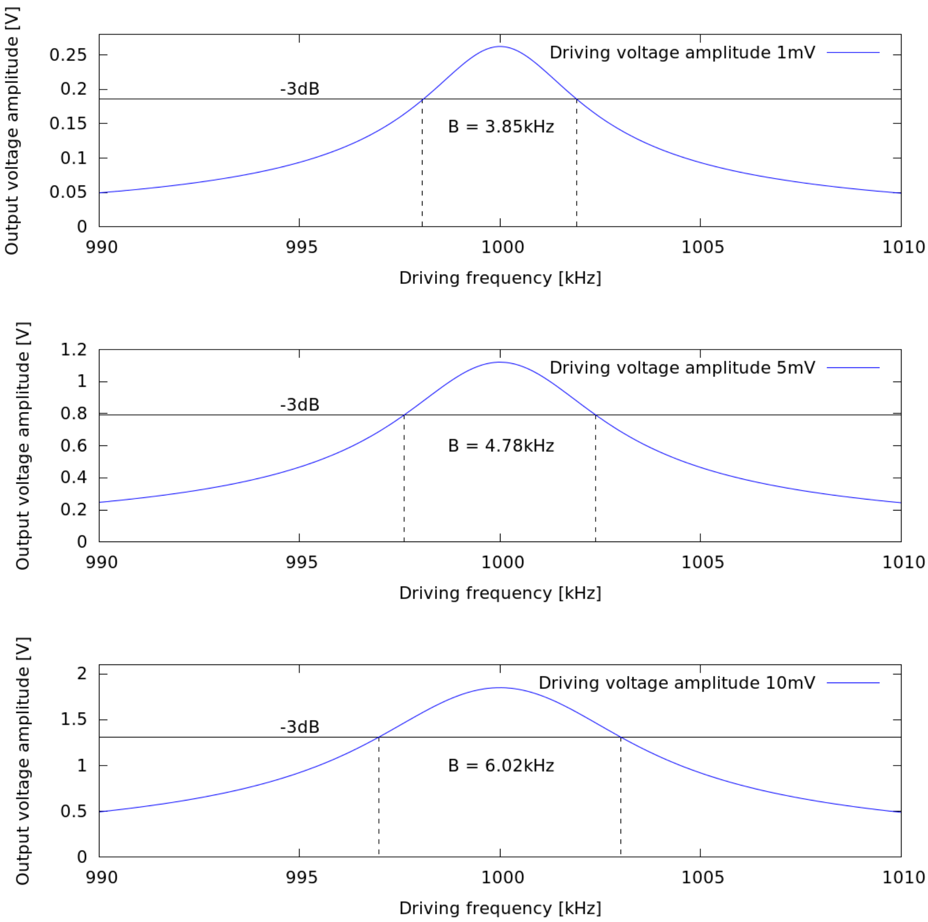

Figure 4: Frequency response curve for different driving voltage amplitudes

However, the oscillating state of a regenerative circuit can be used to demodulate radio signals with suppressed carrier modulation, most notably single sideband (SSB) transmissions. Also, demodulation of regular amplitude modulated signals is still possible in the oscillating state and there are often conflicting reports by vintage radio enthusiasts whether the oscillating state or the non-oscillating state offers better reception. We shall delve into this question a little later in this section. First however, we need to discuss the important issue of injection pulling and injection locking of oscillators by external signals that is fundamental to the oscillating state of regenerative receivers.

Injection Pulling And Injection Locking

If an external signal is applied to an otherwise free-running oscillator, the external signal can under certain conditions synchronize the oscillator so that it becomes not only frequency- but also phase locked with the external signal. For LC oscillators, the conditions under which an external (injected) sinusoidal voltage can synchronize the oscillator have been theoretically examined by R. Adler in the mid 1940s [6]. Adler derived a differential equation for the time dependence of the phase shift between the injected voltage and the output voltage of the oscillator. In the injection locked state where there is a time independent phase angle between the injected voltage and the output voltage of the oscillator this differential equation reduces to

$$\sin(\delta) = 2Q\frac{\Delta f}{f_0}\frac{\hat{U}_{\text{osc}}}{\hat{U}_{\text{inj}}} \tag{7}$$

where δ is the phase angle between the injected voltage and the output voltage of the oscillator, Q is the physical Q-factor of the LC tank, f_0 is the free-running frequency of the oscillator, Δf is the difference between f_0 and the frequency of the injected voltage, $\hat{U}_{\text{osc}}$ is the amplitude of the free-running oscillations and $\hat{U}_{\text{inj}}$ is the amplitude of the injected voltage. Obviously, the above equation also implies the injection locking conditions since $-1 \leq \sin(\delta) \leq 1$.

It should however be stressed that in our circuit model the injection voltage U_{inj} is not the driving voltage U_{d} but the part of the output voltage U_C that occurs at the input of the feedback device due to the presence of the driving voltage U_{d}. Fortunately, deducing U_{inj} from U_{d} is relatively simple. If we take another look at our circuit model, we see that the nonlinearity occurs solely in the feedback device, i.e. $I_{\text{f}} = I_{\text{f}}(U_C)$ and therefore also $U_{\text{f}} = U_{\text{f}}(U_C)$ are nonlinear functions, while the main tank itself is linear. Hence, for all voltages and currents occurring in the main tank, the principle of linear superposition holds true. In particular, the output voltage $U_C(t)$ of the main tank can formally be written as the sum of the output voltage of the tank driven by $U_{\text{f}}(t)$ while $U_{\text{d}} \equiv 0$ and the output voltage of the tank driven by $U_{\text{d}}(t)$ while $U_{\text{f}} \equiv 0$. Obviously, the latter is the injection voltage $U_{\text{inj}}(t)$ we are looking for.

Since we are only interested in a rough estimate of the injection locking range and the locking phase shift δ, we can make the following approximation: Since the injection locking range is typically much smaller than the -3dB bandwidth of the LC tank, we can assume that for injection locking to occur, the driving voltage frequency needs to be well inside the -3dB range around the resonant frequency of the tank[4]. The amplitude $\hat{U}_{\text{inj}}$ of the injection voltage function $U_{\text{inj}}(t)$ is then roughly [7] $\hat{U}_{\text{inj}} \approx Q \cdot \hat{U}_{\text{d}}$ where Q is the physical[5] Q-factor of the tank and $\hat{U}_{\text{d}}$ is the amplitude of the driving voltage function $U_{\text{d}}(t)$. Using this result in equation (7) then yields

[4]determining in good approximation the frequency of the free-running oscillations

[5]since $\hat{U}_{\text{inj}}$ needs to be calculated with $U_{\text{f}} \equiv 0$

$$\sin(\delta) = 2\frac{\Delta f}{f_0}\frac{\hat{U}_{\rm osc}}{\hat{U}_{\rm d}} \tag{8}$$

Let us now use our circuit model, respectively differential equation (4) to numerically explore injection locking. For fixed values of Q, f_0, $\hat{U}_{\rm osc}$ and $\hat{U}_{\rm d}$, the occurrence of injection locking is entirely a matter of the distance Δf between the free-running frequency of the oscillator and the frequency of the driving voltage. Let us now set $b_2 = 0.6$, resulting in $\tilde{R} = -6\Omega$ in the small signal linear approximation to ensure a quick startup of the oscillations. Numerically, we find the free-running amplitude of the oscillations to be $\hat{U}_{\rm osc} = 1.633V_{\rm p}$. Let us now apply a driving voltage of $\hat{U}_{\rm d} = 3{\rm mV}$ whose frequency is lowered from $f_0 + 1300{\rm Hz}$ to $f_0 + 1100{\rm Hz}$ and then finally to $f_0 + 1000{\rm Hz}$. The resulting envelope curves of the oscillator output voltage $U_C(t)$ are shown in figure 5.

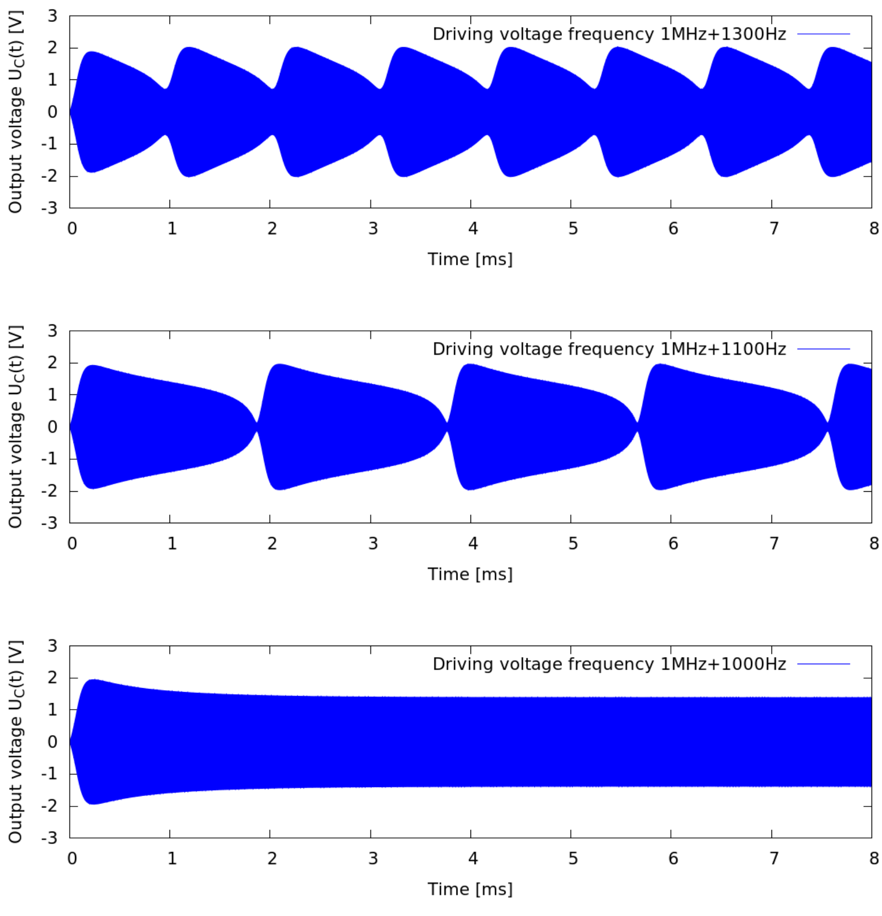

Figure 5: Beat frequency, injection pulling and injection locking

At a driving frequency of $f = f_0 + 1300\text{Hz}$ we can clearly see a distorted beat whose frequency can be determined to be 935Hz which is below a beat frequency of 1300Hz that one might have expected. The reason for this is that even before injection locking occurs, the injected signal pulls the frequency of the oscillator towards it's own frequency. This effect is known as injection pulling [9]. As the frequency of the driving voltage is getting closer to the locking range, the pulling increases and for $f = f_0 + 1100\text{Hz}$ the result is a beat frequency of 526Hz which is about half the beat frequency that would occur without injection pulling of the oscillator. Finally, as the frequency of the driving voltage enters the locking range, the beat disappears as the oscillator is now injection locked to the driving voltage. A closer numerical analysis reveals that locking occurs somewhere around $f = f_0 + 1050\text{Hz}$ which is in quite good agreement with the rough estimate of $f_0 + 919\text{Hz}$ obtained from equation (8).

Carrier Boost And Enhanced Selectivity

The basic question that arises at this point is how the regenerative receiver behaves in the oscillating state when injection locking of the oscillations to the carrier frequency of an AM radio station has occurred. Obviously, in the case of injection locking the original carrier of the received AM signal in the main tank is replaced by the synchronized, locally generated oscillations[6]. These oscillations will typically have a much larger amplitude than the original carrier and will also be phase shifted against it, with the approximate phase shift given by equation (8). Since any noticeable phase shift of the locally generated oscillations towards the original carrier and therefore the upper and lower sidebands of the AM signal will cause distortions and partial mutual cancellations of the sidebands [8], further fine-tuning of the receiver's frequency control must be performed to minimize the phase shift of the locally generated oscillations. Hence, even with the oscillating regenerative receiver there is no "snap-to-station" tuning feature once the injection locking range is reached[7].

Let us therefore assume that the frequency control of the oscillating regenerative receiver has been tuned to not only reach the locking range but also to minimize the phase shift of the locally generated oscillations with respect to the original carrier. In this case the replacement of the original carrier by the locally generated oscillations will lead to a carrier boost effect where the median amplitude of the amplitude modulated output voltage of the main tank is drastically increased. However, since the sidebands of the AM signal remain untouched, the modulation

[6] In a true multiplicative homodyne receiver, the carrier itself is retained and fed into a multiplicative frequency mixing stage along with the output of the local oscillator.

[7] A more advanced homodyne receiver using a phase locked loop oscillator will, however, have this snap-to-station feature for the synchronization range where the local oscillator is locked to a phase shift of close to zero with the carrier.

depth will decrease accordingly. Let us look at an amplitude modulated RF signal of the form

$$s(t) = (1 + m\cos(\omega_{AF}t)) \cdot (\cos(\omega t))$$

where the amplitude of the RF carrier has been normalized to 1V. In the above equation, ω is the frequency of the RF carrier, ω_{AF} is the frequency of a cosine-shaped audio signal[8] and $0 \leq m \leq 1$ is the modulation depth. Using basic trigonometric identities [10], this can be written as

$$s(t) = \cos(\omega t) + \frac{1}{2}m\left(\cos\left((\omega - \omega_{AF})t\right) + \cos\left((\omega + \omega_{AF})t\right)\right)$$

from which the carrier as well as the upper and lower sidebands are readily identified. If we boost the carrier by a factor of $\mu > 0$ while leaving the sidebands untouched, i.e.

$$s(t) = \mu\cos(\omega t) + \frac{1}{2}m\left(\cos\left((\omega - \omega_{AF})t\right) + \cos\left((\omega + \omega_{AF})t\right)\right) \tag{9}$$

we finally end up with [10]

$$s(t) = \mu\left[\left(1 + \frac{m}{\mu}\cos(\omega_{AF}t)\right) \cdot \cos(\omega t)\right] \tag{10}$$

Obviously, boosting only the carrier by a factor of μ results in the median amplitude being increased by this factor, while simultaneously the modulation depth is decreased to m/μ, hence the amplitude of the AF signal borne by the RF signal is not changed. This is also illustrated in figure 6.

We shall now discuss the implications of these results on detection and, in particular, on selectivity. At first glance, boosting the carrier only seems beneficial when the resulting signal is fed into a non-linear envelope detection stage that has a steeper characteristic I-V curve for higher input voltages [9] and will therefore yield higher AF output voltages for carrier boosted RF signals.

However, there's also a far more important benefit of boosting the carrier of an amplitude modulated RF signal. Increasingly crowded AM bands in the 1930s and early 1940s lead to the demand for enhanced selectivity even with cheap receivers. In the search for cost-effective solutions it was discovered that the behavior of AM detectors based on diode or grid-leak detection with low-pass filtering also affect the selectivity of the receiver. A very good introduction to this topic can be found in [2] and the references therein.

[8] By virtue of Fourier's Theorem the following considerations apply to any audio signal.
[9] A typical example of such a detection stage is an anode bend detector.

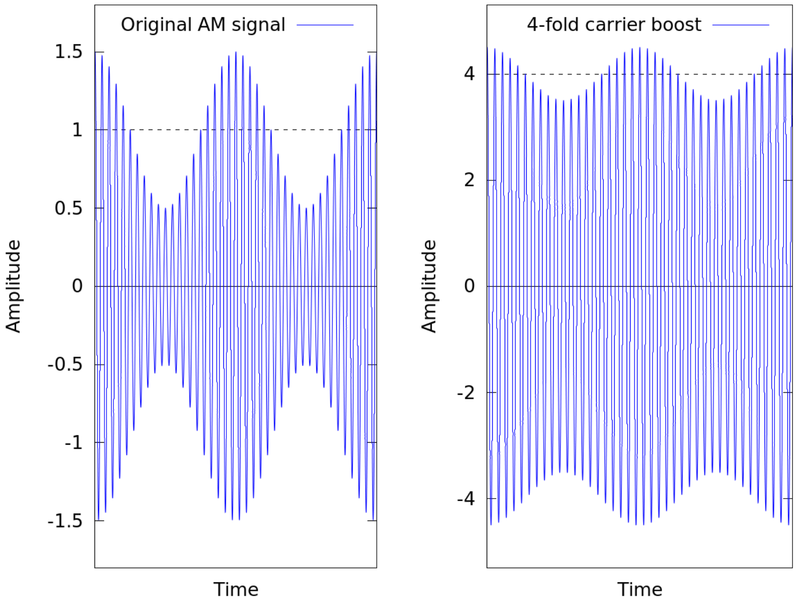

Figure 6: Reduced modulation depth due to carrier boost

It was found that for diode, respectively grid-leak detectors where the low pass filter has a roll-off frequency above the frequency difference between two neighboring AM stations (9kHz in Europe, 10kHz in North America) audible content from neighboring stations is increasingly suppressed as the amplitude of the carrier the receiver is tuned to increases relative to the carrier amplitudes of neighboring stations. In non-oscillating mode, the relative amplitudes of selected and neighboring stations are solely determined by the frequency response curve of the RF input tank. However, going into oscillating mode and thereby boosting the carrier of the selected station can greatly enhance it's amplitude relative to neighboring carriers and hence greatly improve selectivity of the receiver.

At this point it should be stressed that the improvement in selectivity due to carrier boost in an oscillating regenerative receiver as described above is fundamentally different from the improvement in selectivity in a true multiplicative homodyne receiver. In the latter, the original carrier is retained and a multiplicative frequency conversion with a re-created carrier[10] directly down to the audio frequency range is performed. This puts the audio content from neighboring stations in the AF spectrum above half the frequency difference between two neighboring AM stations (again: 9kHz in Europe, 10kHz in North America). These audio frequencies above 4.5kHz, resp. 5kHz then need to be suppressed in the AF amplifier of the receiver.

[10]either from a limiting amplifier or a PLL oscillator

Proper Measurements In Oscillating Mode

The question that arises at this point is how to properly perform output amplitude and bandwidth measurements in the oscillating regenerative circuit that is injection-locked to the carrier of an incoming amplitude modulated RF signal. Simply measuring the output amplitude arising from a single frequency driving voltage as we did in the non-oscillating case will basically lead to a study of the injection locking behavior of the circuit again. However, our goal is to study the behavior of the oscillating regenerative circuit that is already injection-locked to the carrier of an amplitude modulated RF signal.[11] In this case, the "input signal" that we need to focus on are actually the two sidebands of the amplitude modulated RF signal and we are looking for the amplitude and frequency response curve for these sidebands. But how do we extract information on the output voltage arising from the presence of the two sidebands from the overall output voltage?

From the considerations in the previous section we have seen that the reasonable range of operation of the oscillating regenerative receiver is where the phase shift between the RF carrier and the locally generated oscillations is near zero. In this case, and if we take the frequency response curve to be approximately symmetric around the carrier (respectively local oscillations) frequency, the attenuation of the sidebands can readily be obtained from the resulting modulation depth of the output signal. In fact, if we introduce a common sideband attenuation factor of $0 < \gamma \leq 1$ in equation (9), i.e.

$$s(t) = \mu\cos(\omega t) + \frac{1}{2}m\left(\gamma\cos\left((\omega-\omega_{AF})t\right) + \gamma\cos\left((\omega+\omega_{AF})t\right)\right)$$

we immediately obtain

$$s(t) = \mu\left[\left(1+\frac{m\gamma}{\mu}\cos(\omega_{AF}t)\right)\cdot\cos(\omega t)\right]$$

Obviously, the response of the circuit in oscillating and carrier-injection-locked state to the sidebands of the input AM signal can immediately be seen from the modulation depth

$$m_{\text{out}} = m\cdot\frac{\gamma}{\mu} \tag{11}$$

of the output signal and we shall now use m_{out} to study amplitude compression and frequency response of the circuit in oscillating mode.

[11] This mistake has sometimes been made in the early days of radio, see also remark #3 in [11].

Amplitude Compression

From the previous analysis of the non-oscillating state of the regenerative circuit it is reasonable to assume that also in the oscillating state there is an amplitude compression effect on the output voltage U_C. After all, due to the appreciable amplitude of the local oscillations the feedback device will be operated in a region where it provides less feedback than around $U_C = 0$ most of the time. However, we still need to seek for some numeric evidence to support the above argument.

Let's take another look at equation (11) while keeping in mind that μ is proportional to the amplitude $\hat{U}_{\text{osc}}$ of the locally generated oscillations. Hence if there were no amplitude compression, the modulation depth of the output signal m_{out} as a function of $\hat{U}_{\text{osc}}$ would exhibit a rigorous $1/\hat{U}_{\text{osc}}$ dependency even for large oscillation amplitudes. Any amplitude compression will obviously decrease the modulation depth m_{out} of the output signal even further and therefore lead to a decay of m_{out} while $\hat{U}_{\text{osc}}$ increases that will be faster than $1/\hat{U}_{\text{osc}}$. Therefore, we shall plot m_{out} over $\hat{U}_{\text{osc}}$ and analyze it's decay behavior. This has been done in figure 7 for a median driving voltage amplitude of 3mV that is amplitude modulated by a 400Hz sinusoidal audio signal at an initial modulation depth of 50%.

The upper plot shows m_{out} over $\hat{U}_{\text{osc}}$ on a linear scale, while the lower plot uses a logarithmic scale on both axes and compares the actual decay behavior to a $1/\hat{U}_{\text{osc}}$, respectively a $1/\hat{U}_{\text{osc}}^2$ decay. Obviously, the decay behavior starts with $1/\hat{U}_{\text{osc}}^2$ and then drops even faster as $\hat{U}_{\text{osc}}$ increases further. The conclusion, of course, is that for an amplitude range of the locally generated oscillations of 1.6V and above there is a very pronounced amplitude compression effect in our circuit model in the oscillating state.

Frequency Response And Audio Bandwidth

We shall now turn our attention to the frequency response properties of the circuit in it's oscillating and carrier-injection-locked state. As in the previous section, we need to focus on the modulation depth of the output signal and hence the sideband attenuation factor for the amplitude modulated RF input signal to gain correct insight into the frequency response behavior of the circuit. More specific, we shall look at the modulation depth of the output signal m_{out} as a function of the audio frequency ω_{AF} the input signal has been modulated with, since the frequency of the two sidebands is, of course, given by $\omega - \omega_{AF}$ and $\omega + \omega_{AF}$ where ω is the carrier frequency. Due to amplitude compression, we expect the frequency response curve to be more "flat" for larger amplitudes of the locally generated oscillations. This assumption is indeed corroborated by the numerical results presented in figure 8 where again a driving voltage with a median amplitude of 3mV and an initial modulation depth of 50% has been used.

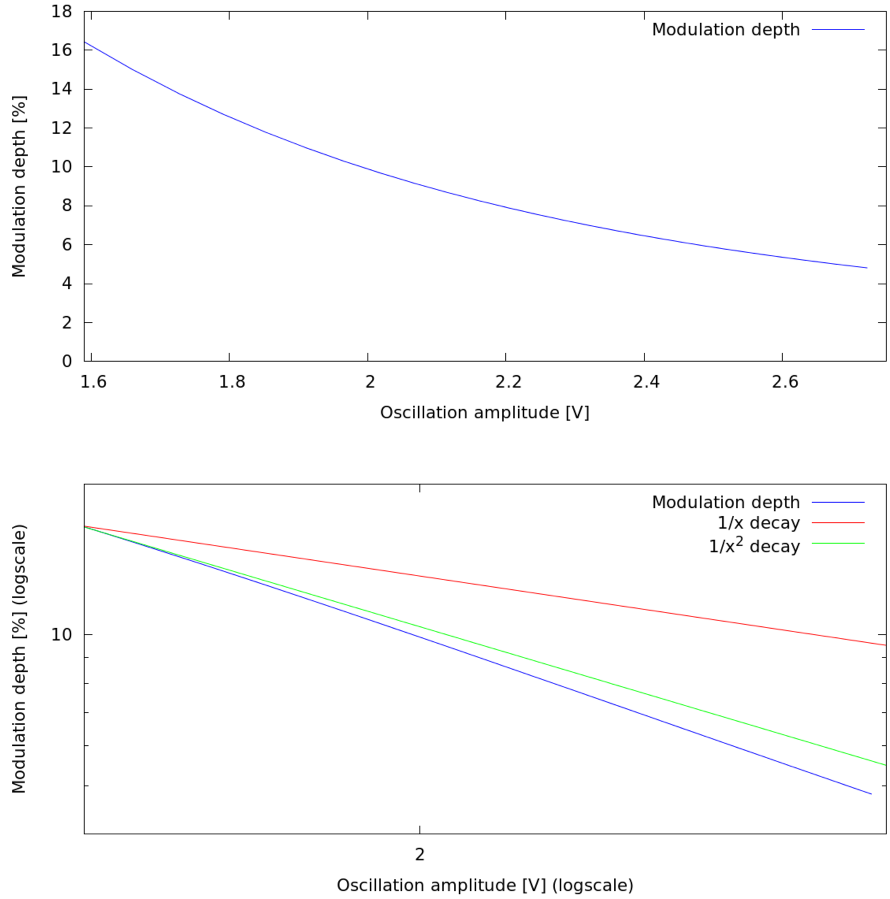

Figure 7: Amplitude compression in oscillating mode

Obviously, for larger amplitudes of the locally generated oscillations, the circuit will exhibit little cut-off for higher audio frequencies that would in non-oscillating mode lead to an adverse selection behavior in crowded AM broadcast bands. Since in oscillating mode however, selectivity is already drastically increased by the carrier boost effect, this augmented audio frequency range can lead to an improved sound quality with the adverse effect on selectivity being barely noticeable.

Conclusions

After numerically analyzing our simple regenerative circuit model with real-time feedback limiting, we are able to draw a few important conclusions: In oscillating mode as well as in non-oscillating mode there is an amplitude compression effect leading to an increase in bandwidth. In non-oscillating mode, this increase

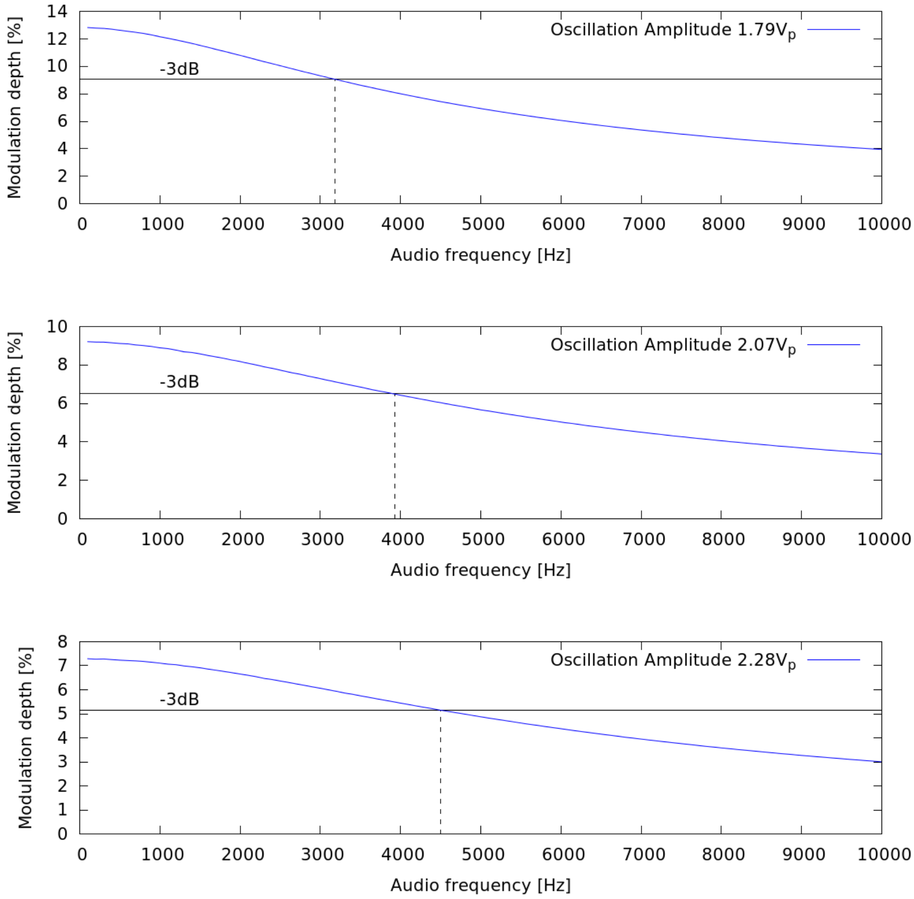

Figure 8: Modulation depth as a function of audio frequency

in bandwidth can adversely affect selectivity and is one of the reasons why it is desirable to start with a high physical Q-factor of the tank that requires less feedback. It has also become obvious that a regenerative circuit in oscillating mode is fundamentally different from a true multiplicative homodyne receiver. In particular, the increase in selectivity in an oscillating regenerative circuit is based on a carrier boost effect in conjunction with certain properties of the envelope detector. Selectivity in a true multiplicative homodyne receiver is, however, obtained by the fact that the audio frequency content of neighboring stations is converted into a frequency range beyond half the frequency distance between carriers and then subsequently suppressed using filters in the audio frequency section of the receiver.

References

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