Calculation Of The Properties of Practical Triodes

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Abstract

The behavior of conventional triodes with helix-shaped grid wires has been extensively treated in the literature on vacuum tube theory. Using several simplifications, in particular pertaining to the influence the grid has on the cathode, the plate current of a triode as a function of the grid and plate voltages can be derived from the laws of motion of electrons in electric fields. The application of the results obtained under these simplifications to practical vacuum tubes however is often problematic. In this paper, an empirically motivated simple modification to the plate current function is presented making the resulting function and the expressions for the triode's transconductance and plate resistance derived from it more applicable to practical vacuum tubes. Usage examples of this modified plate current function along with a discussion of it's accuracy are given for the 12AX7 (ECC83) audio frequency triode.

Plate Current Of A Triode

The behavior of a triode at sufficiently low frequencies where inter-electrode capacitances can be neglected is solely determined by the plate (anode) current I_A as a function of grid voltage U_g and plate (anode) voltage U_A. By virtue of the laws of electron motion in electrostatic fields and making certain simplifications (mostly pertaining to the influence of the grid on the cathode) that might be difficult to achieve in practical vacuum tube designs, the following expression for $I_A(U_g, U_A)$ for an indirectly heated triode (uniform cathode potential) that has **not** been designed for variable transconductance can be derived [1]:

$$I_A(U_g, U_A) = P\left(U_g + \frac{1}{\mu}U_A\right)^{\alpha} \tag{1}$$

Here, P is the so-called perveance of the tube, μ is the amplification factor of the tube (it will later be shown that μ is indeed the open circuit voltage amplification factor) and α is a constant equal to 3/2. The above expression also applies to multi-grid tubes in triode connection provided they are not designed for variable transconductance. An example I_A versus U_g curve with $U_A = 200V$, $\mu = 100$ and $P = 1.7\mathrm{mAV}^{-3/2}$ has been plotted along with data taken from a practical triode (the 12AX7 for $U_A = 200\mathrm{V}$) that has a matching amplification factor and perveance. This plot is shown in figure 1.

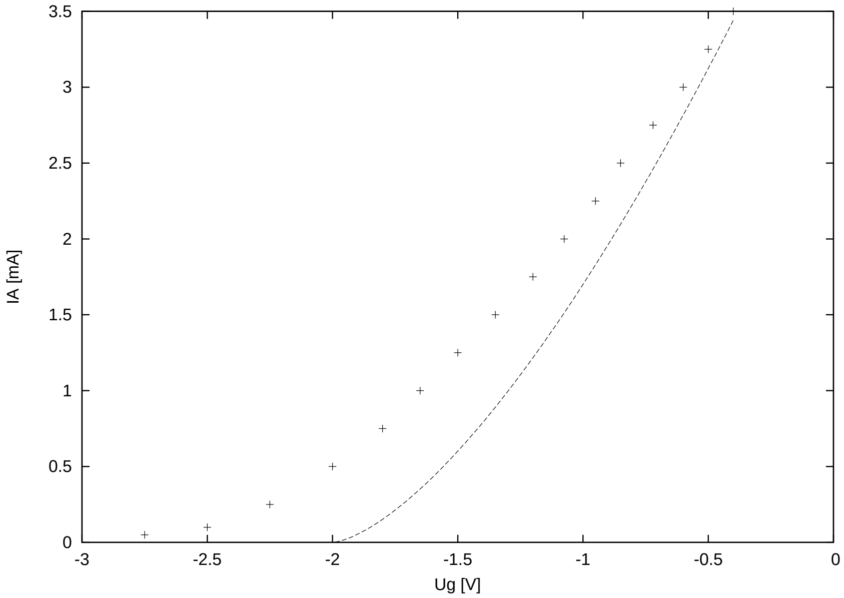

Figure 1: I_A versus U_g curve of a triode

Although the I_A versus U_g curves of such an “ideal grid/cathode” triode resemble the curves the reader is familiar with from various triode data sheets, it fails to accurately provide a curve fitting the $I_A(U_g, U_A)$ behavior of practical triodes. The most obvious reason for this is the relatively sharp cut-off of the plate current at $U_g = -U_A/\mu$ exhibited by the ideal grid/cathode triode that is not present with practical triodes where is takes a much lower grid voltage than $U_g = -U_A/\mu$ for the plate current to vanish. This behavior occurs since the grid of a conventional vacuum tube is usually a wire wound as a helix at a certain distance around the cathode rod (for an example of non-conventional vacuum tubes with different “grid” structures see [4]). When the voltage of the grid wire becomes sufficiently negative against the cathode rod, the parts of the cathode rod that are close to a section of the grid wire get cut off first, while parts of the cathode rod that have a greater distance to the nearest section of the grid wire are still active. It therefore takes a grid voltage much lower than $U_g = -U_A/\mu$ to completely cut the cathode off [2]. Another way to look at this is to regard the plate current that is still present at the theoretical cut-off point $U_g = -U_A/\mu$ as a vestigial current

I_V. We can account for this vestigial current I_V by changing equation (1) to

$$\boxed{I_A(U_g, U_A) = P\left(U_g + \frac{1}{\mu}U_A\right)^{\alpha} + I_V} \tag{2}$$

Also, we allow α to take values other than $\alpha = 3/2$ as long as $\alpha > 1$. The above equation, valid for $U_g \geq -U_A/\mu$, can be used to fit the $I_A(U_g, U_A)$ curves of a typical constant μ triode quite well, provided that the parameters P, I_V and α are being chosen properly. In the following sections of this paper, we will need the plate voltage U_A as a function of grid voltage U_g and plate current I_A. This function $U_A(U_g, I_A)$ can readily be obtained by solving (2) for U_A. The result is:

$$U_A(U_g, I_A) = \mu\left(\frac{I_A - I_V}{P}\right)^{\frac{1}{\alpha}} - \mu U_g \tag{3}$$

We shall further explore the equations given above using the 12AX7 (ECC83) audio frequency vacuum tube as an example. A quite exhaustive data sheet for this tube has been provided by General Electric [3]. The amplification factor of the 12AX7 is given in the data sheet to be $\mu = 100$. The task we are facing is determining the parameters P, I_V and α so that $I_A(U_g, U_P)$ fits the experimentally determined curves given in the data sheet.

Let us start with the vestigial current I_V. Since $I_A = I_V$ for $U_g = -U_A/\mu$ (the theoretical cut-off point) we can extract I_V directly from the set of I_A versus U_g curves for various plate voltages U_A by simply looking up I_A for $U_g = -U_A/100$. The consistent result is $I_V \approx 0.5\text{mA}$.

Let us now turn to the perveance P. Since α is still unknown, we will look at combinations of grid and plate voltages where $U_g + U_A/\mu = 1$ and hence $I_A = P \cdot 1^{\alpha} + I_V = P + I_V$. By looking up I_A for $U_g + U_A/100 = 1$ the perveance is found to be (consistently) $P \approx 1.7\text{mAV}^{-\alpha}$.

Finally, with I_V and P known, α can be determined by solving equation (2) for α, resulting in

$$\alpha = \frac{\ln\left(\frac{1}{P}\left(I_A - I_V\right)\right)}{\ln\left(U_g + \frac{1}{\mu}U_A\right)}$$

and substituting values for U_g, U_A and I_A taken from the curves in the data sheet into the above equation. Of course, combinations where $U_g + U_A/\mu = 1$ can not be used. Also the results will be more accurate for higher values of $(I_A - I_V)/P$ and $U_g + U_A/\mu$. The consistent result is $\alpha \approx 1.2$.

Some I_A versus U_g curves obtained from equation (2) with the parameters P, I_V and α as determined above for the 12AX7 triode are shown in figure 2 along with the corresponding sets of (I_A, U_g) points extracted from the data sheet. It becomes clear that equation (2) describes the behavior of this triode quite well.

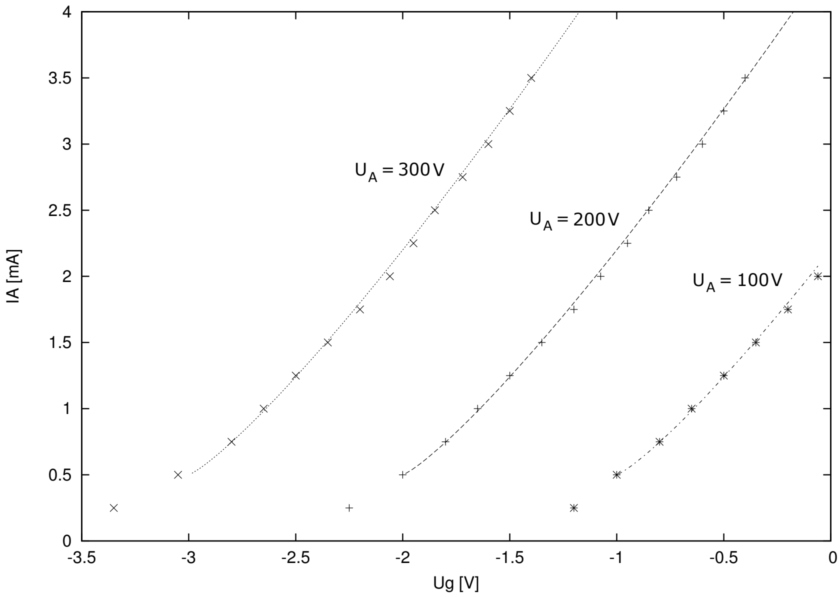

Figure 2: Fitted I_A versus U_g curves for the 12AX7 triode

Amplification Factor

The amplification factor of a triode is defined as $-\Delta U_A/\Delta U_g$ where ΔU_A is the variation of the plate voltage caused by an infinitesimally small variation ΔU_g of the grid voltage while the plate current I_A is kept constant. The negative sign accounts for the fact that the triode when used as a voltage amplifier inverts the input signal. Mathematically speaking, this is nothing else than the partial derivative of the function $U_A(U_g, I_A)$ with respect to U_g, hence

$$-\left(\frac{\Delta U_A}{\Delta U_g}\right)_{I_A=\text{const.}} = -\frac{\partial U_A(U_g, I_A)}{\partial U_g}$$

Substituting $U_A(U_g, I_A)$ from equation (3) into the above definition and performing the partial differentiation simply yields

$$-\left(\frac{\Delta U_A}{\Delta U_g}\right)_{I_A=\text{const.}} = \mu$$

As mentioned earlier, the parameter μ is indeed the amplification factor of the triode.

Transconductance

The transconductance g_m of a triode is defined as $g_m = \Delta I_A/\Delta U_g$ where ΔI_A is the variation of the plate current caused by an infinitesimally small variation ΔU_g of the grid voltage while the plate voltage U_A is kept constant. This can be expressed as the partial derivative of the function $I_A(U_g, U_A)$ with respect to U_g, hence

$$g_m = \left(\frac{\Delta I_A}{\Delta U_g}\right)_{U_A=\text{const.}} = \frac{\partial I_A(U_g, U_A)}{\partial U_g}$$

Substituting $U_A(U_g, I_A)$ from equation (3) into the above definition and performing the partial differentiation gives

$$\boxed{g_m = \alpha P \left(U_g + \frac{1}{\mu}U_A\right)^{\alpha-1}} \qquad (4)$$

In practical applications it is often useful if the transconductance g_m is given as a function of the plate current I_A. Hence, we solve equation (2) for $U_g + U_A/\mu$ and insert the result into the above expression, leaving us with

$$\boxed{g_m = \alpha P \left(\frac{I_A - I_V}{P}\right)^{1-\frac{1}{\alpha}}} \qquad (5)$$

Applying this result to the 12AX7 triode, we can use the values for the parameters P, I_V and α determined earlier for this tube to create a g_m versus I_A plot and compare it to a set of (g_m, I_A) points extracted from the tube's data sheet. The result is shown in figure 3.

The breakdown of the approximation provided by the curve obtained from equation (5) to the experimental values at $I_A = I_V = 0.5\text{mA}$ is expected, since equation (2) on which equation (5) is based upon is not valid beyond this point. Otherwise, the curve approximates the experimental values quite well although there is a systematic error of up to $\Delta g_m \approx 0.2\text{mS}$ that can be attributed to the fact that equation (2) does not fully cover the complicated behavior of practical triodes.

Plate Resistance

The plate (anode) resistance R_A of a triode is defined as $R_A = \Delta U_A/\Delta I_A$ where ΔU_A is the variation of the plate voltage caused by an infinitesimally small variation ΔI_A of the plate current while the gate voltage U_g is kept constant. Mathematically, this is given by the partial derivative of the function $U_A(U_g, I_A)$ with respect to I_A, therefore

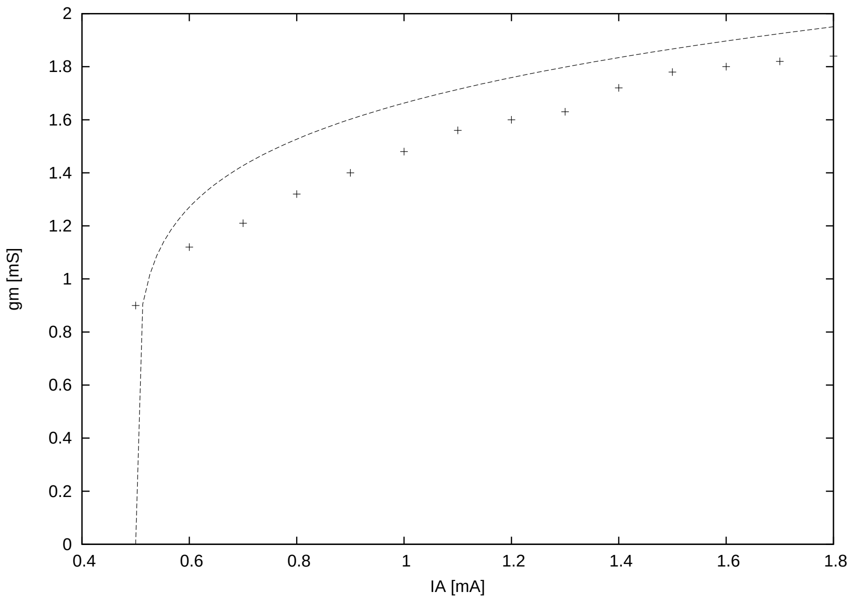

Figure 3: Transconductance curve for the 12AX7 triode

$$R_A = \left(\frac{\Delta U_A}{\Delta I_A}\right)_{U_g=\text{const.}} = \frac{\partial U_A(U_g, I_A)}{\partial I_A}$$

Substituting $U_A(U_g, I_A)$ from equation (3) into the above definition and performing the partial differentiation leads to

$$\boxed{R_A = \frac{\mu}{\alpha P}\left(\frac{I_A - I_V}{P}\right)^{\frac{1}{\alpha}-1}} \tag{6}$$

For the sake of completeness, we can express the plate resistance R_A as a function of grid voltage U_g and plate voltage U_A by solving equation (2) for $(I_A - I_V)/P$ and inserting the result into the above equation providing us with

$$\boxed{R_A = \frac{\mu}{\alpha P}\left(U_g + \frac{1}{\mu}U_A\right)^{1-\alpha}} \tag{7}$$

Applying this result again to the 12AX7 triode, we can once more use the values for the parameters P, I_V and α determined earlier for this tube to create an R_A versus I_A plot and compare it to a set of (R_A, I_A) points extracted from the tube's data sheet. The result is shown in figure 4.

Again, the breakdown of the approximation provided by the curve obtained from equation (6) to the experimental values at $I_A = I_V = 0.5\text{mA}$ is expected, since

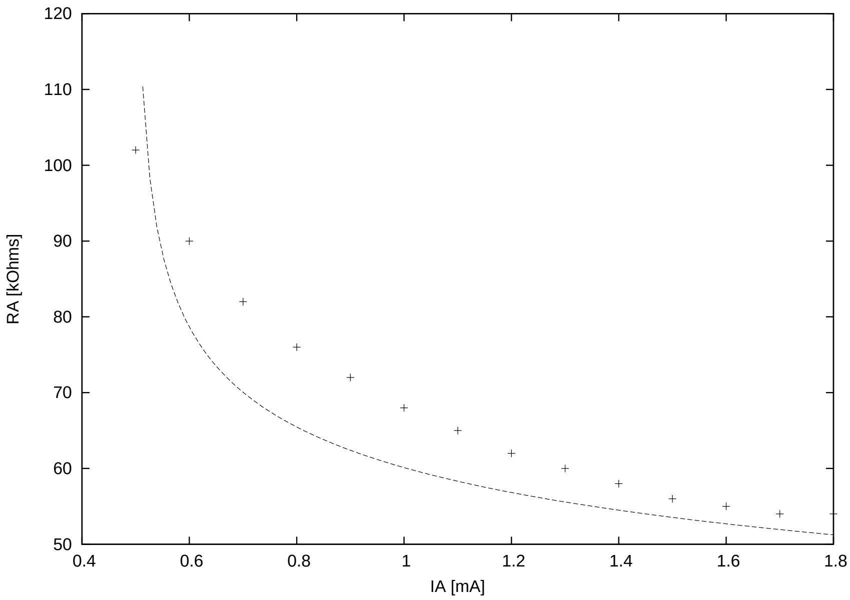

Figure 4: Plate resistance curve for the 12AX7 triode

equation (2) on which equation (6) is based upon is not valid beyond this point. Otherwise, the curve approximates the experimental values quite well although there is a systematic error of up to $\Delta R_A \approx 10\text{k}\Omega$ that, as in the previous section, can be attributed to the fact that equation (2) does not fully cover the complicated behavior of practical triodes.

References

[1] Karl R. Spangenberg, *Vacuum Tubes, MacGraw-Hill*, 1948

[2] http://www.john-a-harper.com/tubes201

[3] General Electric 12AX7 Twin Triode Data Sheet, June 1953

[4] Joseph Sousa, http://www.radiomuseum.org/forum/russian_subminiature_tubes.html