Time-domain modeling and simulation of power supplies

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Summary

The purpose of this note is to describe the Hammerstein model that can be used to simulate the dynamical behavior of the power supply implemented in some radio sets designed in the ’30s when the cost of high voltage paper capacitors was a primary limiting factor suggesting the use of more complex tuned circuits leading to remarkably low ripple levels even using a modest total capacitance; for comparison purposes also the simpler model of traditional power supplies is deduced. The note describes the interaction between the iterative computation required in simulating the non linear part of the Hammerstein model and the simulation of the linear dynamic part with sufficient detail to allow an easy implementation in the Matlab environment whose powerful linear algebra tools reduce the whole simulation code to a modest number of lines. The use of time–domain models allows a great degree of flexibility in that it allows analyzing not only the steady-state behavior but also transients (for instance, the current in the rectifier starting from completely discharged capacitors) and, if desired, the response to non sinusoidal and/or non periodical inputs.

The considered circuits

The tuned power supply considered in the following is reported in Figure 1. The term “tuned” has been introduced to distinguish this kind of power supply from standard designs like that reported in Figure 2. By comparing these designs, it can be observed that the tuned power supply requires an

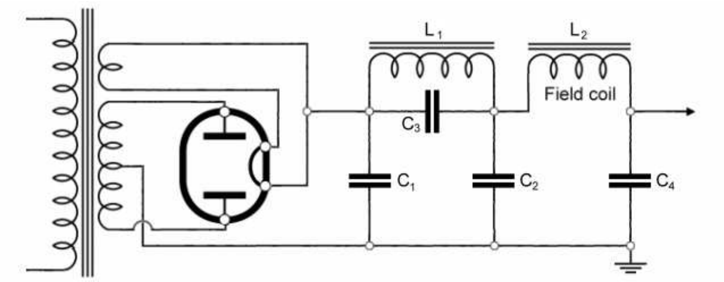

Figure 1 - Tuned power supply implemented in some receivers of the ’30s

additional inductor (L_1) and an additional capacitor (C_3) connected across L_1. The original designs include the resistor used to generate the negative bias for the grid of the output power tube; it has ben omitted here because it does not play any significant role in the context of our ripple-oriented analysis.

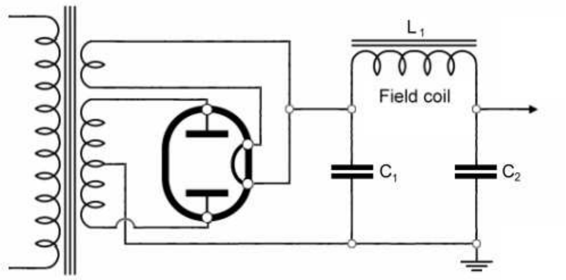

Figure 2 - Standard power supply design

The equivalent circuit of the tuned power supply shown in Figure 3 shows that this non linear system can be described by means of an Hammerstein model, i.e. as the cascade connection of a non linear algebraic model driven by the waveform shown in Figure 5 and of a linear dynamic part.

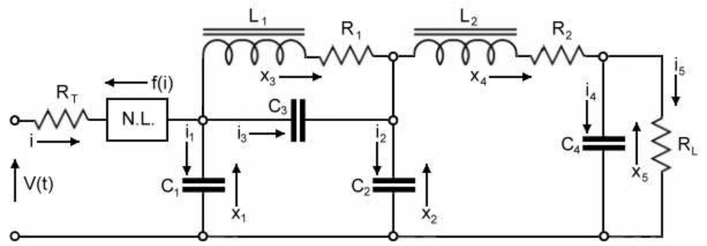

Figure 3 - Equivalent circuit of the tuned power supply

Remark 1 - From the System Theory point of view it can be observed that the considered system is not in *minimal form* in that the same transfer function can be obtained by means of a lower number of components. In fact, making reference to state-space models to describe linear networks, it is well known that the simplest choice for state variables concerns the voltages across capacitors and the currents through inductors and, by following this rule, six state variables would be needed. In the circuit in Figure 3, however, the capacitor C_3 is connected across C_1 and C_2 so that the voltages across these capacitors (denoted as x_1 and x_2) define also the voltage across C_3; consequently five state variables are, in this case, sufficient. As a general engineering rule it would be advisable to design only circuits in minimal form because this corresponds to the use of the minimal number of components; in some cases, however, circuits that do not follow this rule can be optimal from other points of view like reliability, component availability and, as was probably the case in the studied circuit, performance to cost ratio.

The model to be used for simulation will be deduced in the time domain instead than in the frequency domain; this allows the possibility of studying not only the steady-state behavior of the system but also its transients as well as the response to any kind of input (periodic or not).

As a first step we will deduce the continuous-time model of the linear part of the Hammerstein model; the input is the current $i(t)$ flowing through the rectifier and the output is the voltage, x_5, across C_4. Making reference to the state variables and to the currents indicated in Figure 3 it is possible to write the following relations

$$i = i_1 + i_3 + x_3 \tag{1}$$

$$i_1 = C_1\,(dx_1/dt) \tag{2}$$

$$i_3 = C_3\,(dx_1/dt - dx_2/dt) \tag{3}$$

$$x_1 - x_2 = L_1(dx_3/dt) + R_1 x_3 \tag{4}$$

$$i_2 = C_2\,(dx_2/dt) \tag{5}$$

$$x_3 + i_3 = x_4 + i_2 \tag{6}$$

$$x_2 - x_5 = L_2(dx_4/dt) + R_2 x_4 \tag{7}$$

$$x_4 = i_4 + i_5 \tag{8}$$

$$i_4 = C_4\,(dx_5/dt) \tag{9}$$

$$i_5 = x_5/R_L\ . \tag{10}$$

By eliminating i_1, i_2, i_3, i_4 and i_5 we obtain the following first-order linear differential equations in the state components $x_1,\dots,x_5$.

$$dx_1/dt = ((C_1C_3-\alpha)/\alpha(C_1+C_3))\,x_3 - (C_3/\alpha)\,x_4 + ((\alpha+C_3^2)/\alpha(C_1+C_3))\,i(t) \tag{11}$$
$$dx_2/dt = (C_1/\alpha)\,x_3 - ((C_1+C_3)/\alpha)\,x_4 + (C_3/\alpha)\,i(t) \tag{12}$$
$$dx_3/dt = (1/L_1)\,x_1 - (1/L_1)\,x_2 - (R_1/L_1)\,x_3 \tag{13}$$
$$dx_4/dt = (1/L_2)\,x_2 - (R_2/L_2)\,x_4 - (1/L_2)\,x_5 \tag{14}$$
$$dx_5/dt = (1/C_4)\,x_4 - (1/R_LC_4)\,x_5 \tag{15}$$

where $\alpha = (C_1+C_3)(C_2+C_3)-C_3^2$. These relations define the continuous-time state-space model

$$dx/dt \;=\; Ax(t)+Bu(t) \tag{16}$$
$$y(t) \;=\; Cx(t) \tag{17}$$

where $x(t)=(x_1(t),\dots,x_5(t))^T$ denotes the state vector, A is the system dynamical matrix, whose entries are defined by relations (11)-(15), B is the input distribution matrix whose entries are equally defined by (11)-(15), C is the output distribution matrix given by $C=[0\;0\;0\;0\;1]$, $u(t)=i(t)$ is the input and $y(t)=x_5(t)$ is the output.
For simulation purposes it is now useful to construct a discrete-time model associated with a selected sampling interval Δt; the equivalent discrete model is

$$x(t+1) \;=\; A_d\,x(t)+B_d\,u(t) \tag{18}$$
$$y(t) \;=\; C_d\,x(t) \tag{19}$$

where

$$A_d \;=\; e^{A\Delta t} \tag{20}$$
$$B_d \;=\; A^{-1}(A_d-I)\,B \tag{21}$$
$$C_d \;=\; C\;. \tag{22}$$

The algebraic non linear model linking the input current to the input voltage $V(t)$ to the input current $i(t)$ is described by the relation

$$i(t) = (V(t)-x_1(t)-f(i))/R_T \tag{23}$$

where $f(i)$ is the non linear function relating the current through the rectifier (single plate) to the voltage drop between anode and cathode (filament). Solving (23) requires a simple iterative procedure that, for a greater computational efficiency, could start, at time t, from $i(t-1)$ and introduce variations until (23) is satisfied within a selected tolerance.

Remark 2 - The function $f(i)$ can be approximated at any desired accuracy level with a polynomial in i whose coefficients can be obtained by means of standard Least Squares from some points of the V/i curve of the considered rectifier. A third order polynomial and 5-6 points are more than sufficient to obtain a very good accuracy with most rectifiers.

Simulation procedure

The construction of the matrices A, B and C of the state-space model is straightforward once that the values of the circuit components have been selected. The computation of the matrices A_d, B_d and C_d of the discrete time model requires the choice of the sampling interval Δt and the computation of the exponential of A, $e^{A\Delta t}$ that can be performed by means of a single Matlab instruction (`expm`). A set of pairs V/i for the computation of $f(i)$ can be deduced from the datasheet of the considered rectifier or directly measured when not reported (in fact the V/i curve is not always present in the datasheets of rectifiers).

The subsequent computation of the interpolating polynomial can be performed by means of the Matlab `interpolate` command or also, for those who are familiar with Least Squares and with the use of the pseudoinverse of a matrix, by means of the `pinv` command. Construct also the vector of input samples $V(t)$ with the input values selected on the basis of Δt and of the considered simulation interval. After these preliminary steps, whose implementation requires less time than their description, the simulation of the circuit can be performed according with the following steps

1. Set $t = 0$ and select the desired initial state $x(0)$ (for instance $x(0) = (0\ 0\ 0\ 0\ 0)^T$.
2. Compute $i(t)$ by means of a search procedure applied to relation (23). This can be done either by using sophisticated Matlab optimization procedures (like `fminsearch`) or also in a very crude direct way since it will require only a very limited number of steps that will not penalize the whole procedure.
3. Use the discrete model (18)-(19) to compute $x(t+1)$ and $y(t+1)$.
4. Increase t and repeat steps 2 and 3 for all values of t in the simulation interval.

Note that, if the input sequence concerns only one period (like in Figure 5) and the initial state is zero, several simulation cycles (e.g. 40-60) are necessary to reach a steady state condition; of course the initial state at every cycle will be the final one of the previous cycle.

A simulation example

The simulation reported below refers to the following values: $R_T = 400\,\Omega$, $C_1 = 4\,\mu F$, $C_2 = 2\,\mu F$, $C_3 = 0.32\,\mu F$, $C_4 = 2\,\mu F$, $L_1 = 2H$, $R_1 = 220\,\Omega$, $L_2 = 20H$, $R_2 = 1800\,\Omega$, $R_L = 5500\,\Omega$. These values do not correspond to any specific set but are typical of the considered class of power supplies. The function $f(i)$ that has been used describes the type 80 rectifier (widely used in the '30s); its shape is reported in Figure 4.

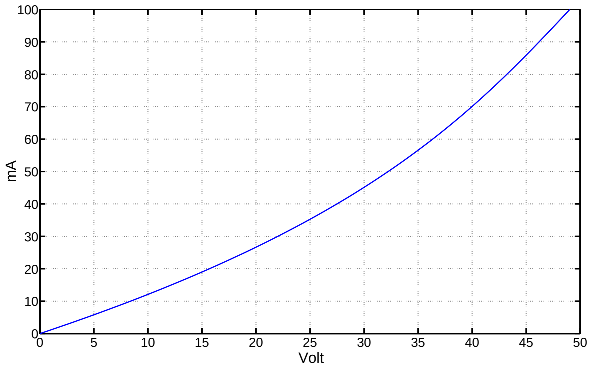

Figure 4 - Order 3 polynomial interpolation of the 80 characteristics

The value of the high voltage has been assumed to be $320V$ and the selected sampling interval is $\Delta t = 0.1$ ms so that the input sequence $V(t)$ contains 200 samples for every (line) period (Figure 5). It can be observed that the selected sampling frequency is 100 times the minimal sampling frequency required by the Shannon theorem for a 50 Hz sinusoid and this assures a large margin in the correct spectral reproduction of all involved signals.

Starting with $x(0) = (0\ 0\ 0\ 0\ 0)^T$, a steady state condition is reached after approximately 40 periods (0.8 s). The input current is reported in Figure 6 while the voltages across C_1 and C_2 are reported in Figures 7 and 8. The current through L_1 and L_2 is reported in Figures 9 and 10 and, finally, the voltage across C_4 is reported in Figure 11.

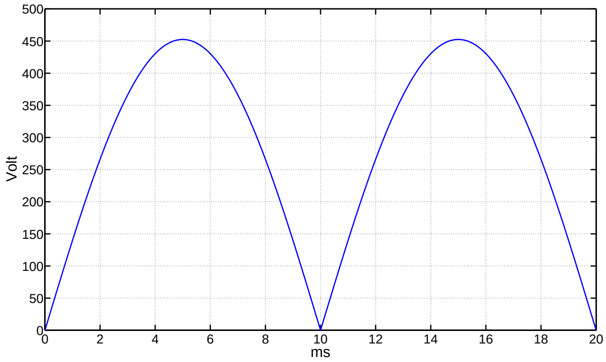

Figure 5 - Input sequence (200 samples) for every line period

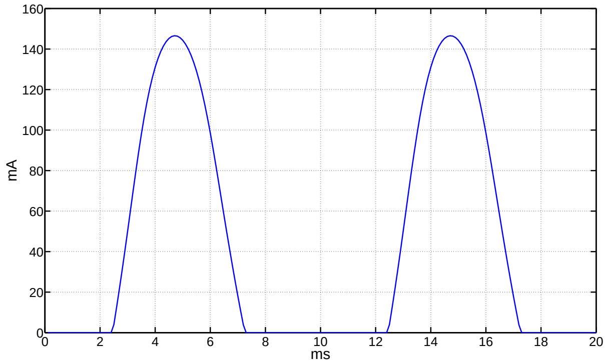

Figure 6 - Input current

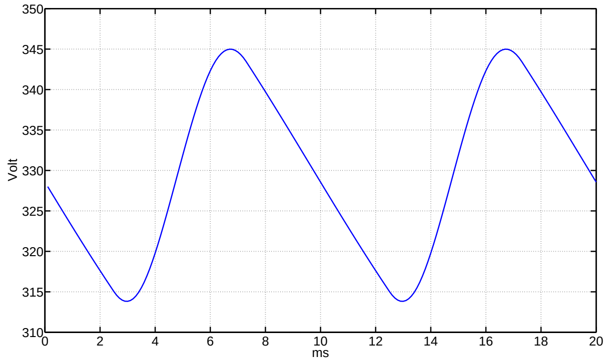

Figure 7 - Voltage across C_1

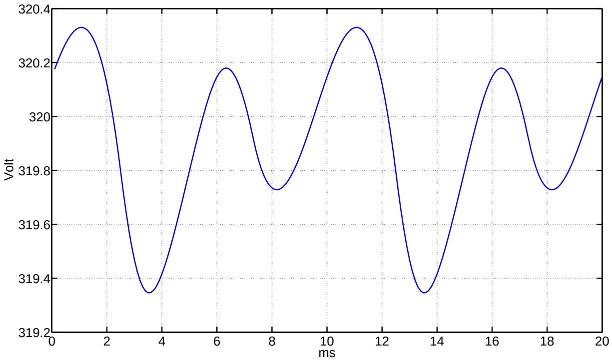

Figure 8 - Voltage across C_2

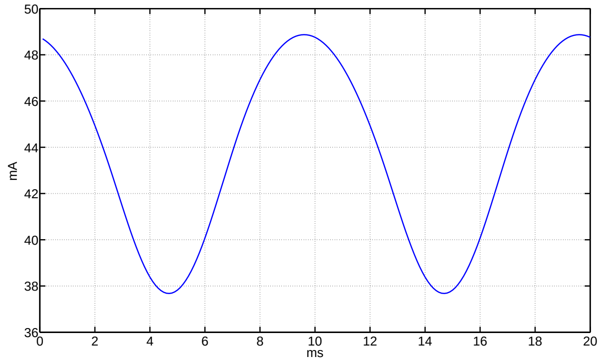

Figure 9 - Current through L_1

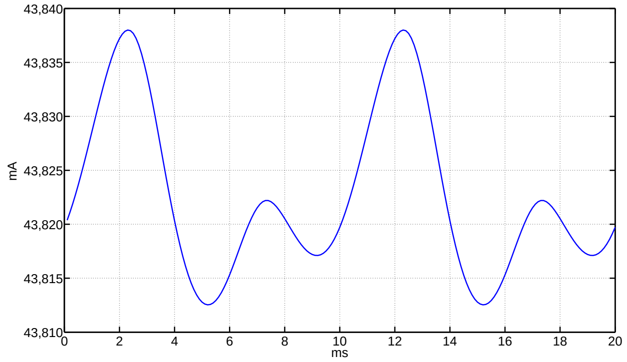

Figure 10 - Current through L_2

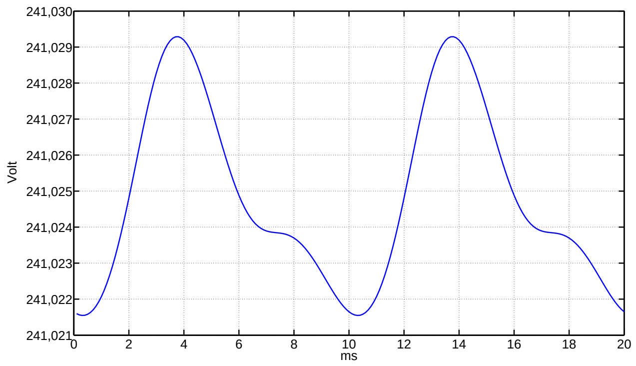

Figure 11 - Voltage across C_4

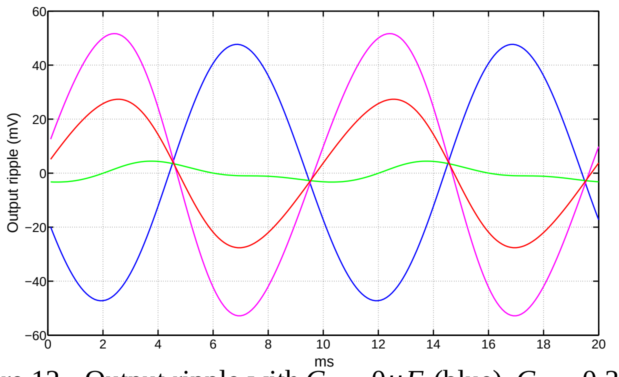

Figure 12 - Output ripple with $C_3 = 0\mu F$ (blue), $C_3 = 0.32\mu F$ (green), $C_3 = 0.6\mu F$ (red), and $C_3 = 1\mu F$ (magenta)

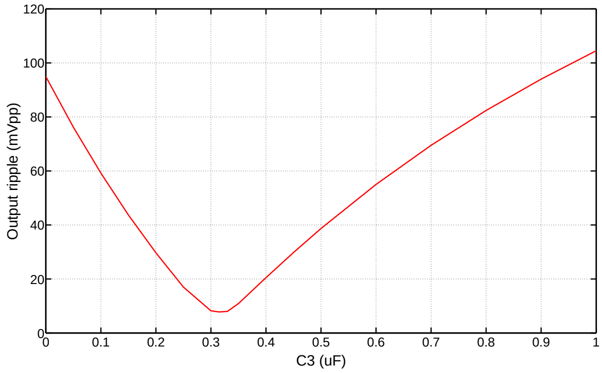

Figure 13 - Output ripple (mVpp) for values of C_3 ranging from 0 to 1 μF

It is interesting to observe that the ripple is quite low (7.7 mVpp) but it increases when the value of C_3 deviates from the optimal value used in the simulation ($0.32\mu F$). Figure 12 shows that for $C_3 = 0\mu F$ we get 94.9 mVpp, for $C_3 = 0.6\mu F$ 55.0 mVpp and for $C_3 = 1\mu F$, 104.5 mVpp; note the inversion of the ripple phase crossing the optimal value of C_3. Figure 13 reports the value of the ripple (mVpp) for values of C_3 ranging from 0 to $1\mu F$.

Comparison with standard designs

The equivalent circuit of the standard power supply shown in Figure 2 is reported in Figure 14 and can be described by means of an Hammerstein model whose algebraic non linear part is identical to the already described one.

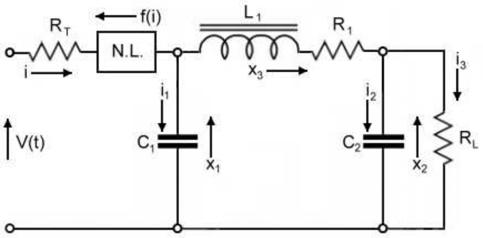

Figure 14 - Equivalent circuit of a standard power supply

The relations that can be written are now the following

$$i = i_1 + x_3 \tag{24}$$
$$i_1 = C_1\,(dx_1/dt) \tag{25}$$
$$x_1 - x_2 = L_1(dx_3/dt) + R_1 x_3 \tag{26}$$
$$x_3 = i_2 + i_3 \tag{27}$$
$$i_2 = C_2\,(dx_2/dt) \tag{28}$$
$$i_3 = x_2/R_L\;. \tag{29}$$

By eliminating i_1, i_2 and i_3 we obtain immediately the following simple first-order linear differential equations in the three state components

$$dx_1/dt = -x_3/C_1 + (1/C_1)\,i(t) \tag{30}$$
$$dx_2/dt = -(1/C_2R_L)\,x_2 + (1/C_2)\,x_3 \tag{31}$$
$$dx_3/dt = (1/L_1)\,x_1 - (1/L_1)\,x_2 - (R_1/L_1)\,x_3 \tag{32}$$

that define the entries of the matrices A and B of the continuous-time model (16)-(17). The output distribution matrix is now $C = [0\ 1\ 0]$ and all remaining steps are the same already described for the previous model. To compare the performance of a standard power supply with that of the tuned one, the simulation that has been performed assumes $C_1 = C_{1t}$, $C_2 = C_{2t} + C_{4t}$, $L_1 = L_{2t}$, $R_1 = R_{2t}$ and the same values for $V(t)$ and R_L (the suffix t refers to the components of the tuned power supply). The input current, the voltage across C_1, the current through L_1 and the voltage across C_2 are reported in Figures 15, 16, 17 and 18; the ripple is now 232.3 mVpp, i.e. 30 times larger than that obtained with the tuned design and the same value of total high voltage capacitance ($8\mu F$). It can be of interest to evaluate the total capacitance required by the standard design to reach the same performance as the tuned power supply; by keeping $C_1 = C_2$ it can be seen that the same performance is reached for $C_1 = C_2 = 22\mu F$ (7.6 mVpp) i.e. for a total capacitance of $44\mu F$.

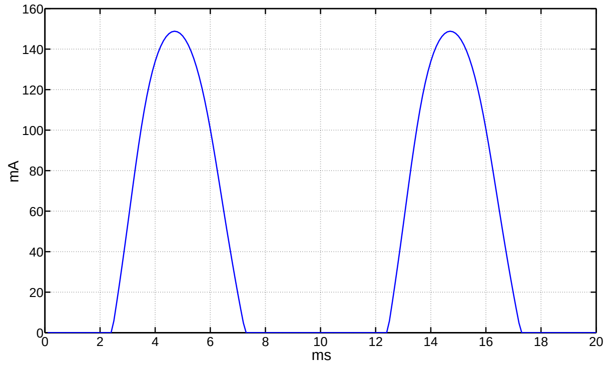

Figure 15 - Input current ($C_1 = C_2 = 4\mu F$)

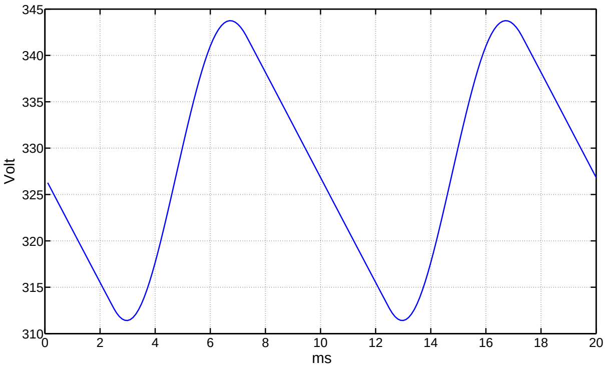

Figure 16 - Voltage across C_1 ($C_1 = C_2 = 4\mu F$)

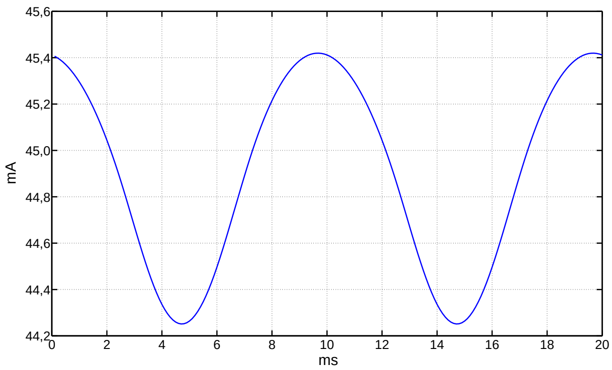

Figure 17 - Current through L_1 ($C_1 = C_2 = 4\mu F$)

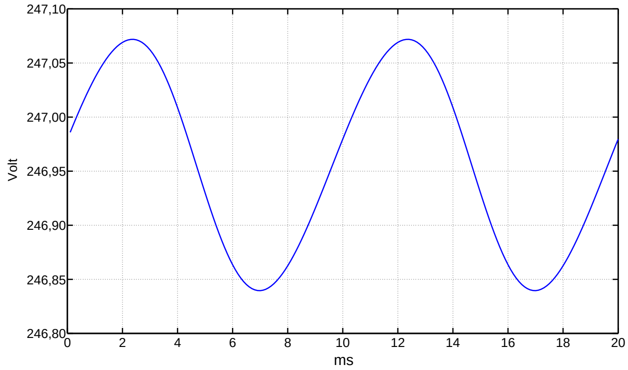

Figure 18 - Voltage across C_2 ($C_1 = C_2 = 4\mu F$)

Concluding remarks

The time-domain simulations, based on Hammerstein models, of the tuned and standard power supplies that have been carried out show that tuned designs can give excellent results in terms of output ripple even with a limited amount of total capacitance at the price of an additional inductor and of a, comparatively small, tuning capacitor. Their use in the ’30s when non polarized large and expensive high voltage capacitors were used constituted a clever solution to a technical and economic problem; the subsequent introduction of electrolytic capacitors has led to prefer standard designs that do not need any auxiliary inductor besides the loudspeaker field coil.